

MATHEMATICS GRADE 3					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Policy		The Level 1 student has not yet reached the approaching proficiency level in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student consistently performs below the standard for the grade level/course, is inconsistently able to access grade-level content, and only engages with higher-order thinking skills with extensive support.	The Level 2 student is approaching proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student generally performs slightly below the standard for the grade level/course, is sometimes able to access grade-level content, and engages in higher-order thinking skills with some independence and support.	The Level 3 student is proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student regularly performs at the standard for the grade level/course, is able to access grade-level content, and engages in higher-order thinking skills with independence and minimal support.	The Level 4 student is highly proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student regularly performs independently or significantly above the standard for the grade level/course, is able to access above grade-level content, and engages in higher order-thinking skills independently.
		The Level 1 Student	The Level 2 Student	The Level 3 Student	The Level 4 Student
Number and Operations					
Range	3.NO.CC.1	Reads and writes numbers up to 1,000 including standard, word, and expanded forms.	Reads and writes numbers up to 10,000 using objects or visual representations including standard and word form.	Reads and writes numbers up to 10,000 using objects or visual representations including standard, word, and expanded form.	Reads and writes numbers up to 10,000 including standard, word, and expanded form.
Range	3.NO.NBT.1	Compares two four-digit numbers using visual representations or words.	Compares two four-digit numbers using symbols.	Compares two four-digit numbers using symbols and justifies comparisons based on the values of thousands, hundreds, tens, and ones.	Compares two four-digit numbers in word, standard, or expanded form using symbols and justifies comparisons based on the values of thousands, hundreds, tens, and ones.
Range	3.NO.NBT.2	Uses place value understanding to round a two-digit number to the nearest 10.	Uses place value understanding to round a three-digit number to the nearest 100.	Uses place value understanding to round whole numbers (up to 1,000) to the nearest 10 or 100.	Uses rounding strategies in authentic situations with whole numbers up to 1,000.
Range	3.NO.NBT.3	Adds and subtracts two-digit numbers using strategies and algorithms based in place value, properties of operations, and/or the relationship between addition and subtraction.	Adds and subtracts numbers within 1,000 using visual models or support.	Adds and subtracts numbers presented in vertical or horizontal orientation within 1,000 using strategies and algorithms based in place value, properties of operations, and/or the relationship between addition and subtraction.	Adds and subtracts numbers presented in vertical or horizontal orientation within 1,000; explains the method used in finding the sum or difference; and recognizes, identifies an error, and shows the correct answer.
Range	3.NO.NBT.4	Skip counts by 10 or repeated addition to multiply single-digit whole numbers by multiples of 10 within 100.	Skip counts or uses grouping strategies to multiply single-digit whole numbers by multiples of 10 within 100.	Multiplies single-digit whole numbers presented in vertical or horizontal orientation by multiples of 10 within 100.	Multiplies single-digit whole numbers presented in vertical or horizontal orientation by multiples of 10 within 100 using strategies based on place value and properties of operations and shows product using multiple strategies.
Range	3.NO.NF.1	Recognizes two-dimensional figures that have been partitioned into parts with equal areas and expresses the area as a unit fraction of the whole (with denominators of 6 or 8).	Partitions two-dimensional figures into parts with equal areas and expresses the area as a unit fraction of the whole (with denominators of 6 and 8).	Partitions two-dimensional figures into equal areas, expresses the area of each part as a unit fraction (with denominators of 6 or 8) of the whole, and describes the parts using the language of sixths, eighths, a sixth of, an eighth of, $\frac{\_}{\_}$ out of 6, and $\frac{\_}{\_}$ .	Partitions shapes in multiple ways into parts with equal areas, expresses the area as a unit fraction of the whole using language such as eighths or an eighth of, and applies understanding of unit fractions to authentic situations and problems.
Range	3.NO.NF.2	Identifies the numerator and denominator of a fraction on a partitioned number line where the increments are equal to the denominator.	Identifies or represents the fraction on a partitioned number line where the increments are equal to the denominator.	Represents a fraction on a number line by partitioning into equal parts.	Represents a set of fractions with unlike denominators on a number line by partitioning into equal parts.
Range	3.NO.NF.3	Represents equivalent fractions using denominators of 2, 4, and 8 using visual representations.	Represents equivalent fractions using denominators of 2, 3, 4, 6, and 8 using a visual representations.	Represents equivalent fractions using denominators of 2, 3, 4, 6, and 8 using visual representations and number lines.	Represents equivalent fractions (including fractions greater than 1 whole) using denominators of 2, 3, 4, 6, and 8 and explains why the fractions are equivalent using a visual model.
Range	3.NO.NF.4	Recognizes and expresses fractions that are equivalent to 1.	Recognizes and expresses fractions that are equivalent to whole numbers.	Recognizes fractions that are equivalent to whole numbers and expresses whole numbers as fractions.	Recognizes fractions that are equivalent to whole numbers and expresses whole numbers as fractions and explains why the fractions are equivalent.

MATHEMATICS GRADE 3					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Range	3.NO.NF.5	Compares two fractions of the same whole with the same denominator (limited to 2, 3, 4, 6, and 8) and records results using symbols.	Compares two fractions of the same whole with the same numerator (denominators limited to 2, 3, 4, 6, and 8) and records results using symbols.	Compares two fractions of the same whole each with a value less than one whole that have the same numerator or same denominator (limited to 2, 3, 4, 6, and 8) using symbols by reasoning about their size.	Applies reasoning strategies to solve authentic or mathematical problems requiring comparison of two fractions of the same whole each with a value less than one whole that have the same numerator or same denominator (limited to 2, 3, 4, 6, and 8) using symbols by reasoning about their size.
Algebraic Reasoning					
Range	3.AR.OA.1	Multiplies single-digit numbers presented in vertical or horizontal orientation using a variety of strategies and supports.	Multiplies and divides all single-digit numbers presented in vertical or horizontal orientation using a variety of strategies and supports.	Using mental strategies, multiplies and divides basic facts within 100 and with automaticity for facts up to 5×5 and 10s facts.	Flexibly multiplies and divides numbers within 100 and with automaticity for facts up to 5×5 and 10s facts; analyzes and explains errors and solutions.
Range	3.AR.OA.2	Applies the properties of operations to solve multiplication equations presented in vertical or horizontal orientation.	Applies the properties of operations to solve multiplication and division equations presented in vertical or horizontal orientation.	Applies the properties of operations as strategies to solve multiplication and division equations presented in vertical or horizontal orientation and justifies thinking.	Applies multiple strategies of operations within a word problem and justifies thinking.
Range	3.AR.OA.3	Solves two-step authentic word problems using addition and subtraction within 500 with simple context and concrete objects or visual representations.	Solves two-step authentic word problems using addition and subtraction within 1,000 with simple context and visual representations (with the unknown in a variety of positions).	Solves two-step authentic word problems using addition and subtraction within 1,000 including equations with a letter as an unknown quantity (with the unknown in a variety of positions).	Creates and solves two-step authentic word problems using addition and subtraction within 1,000.
Range	3.AR.OA.4	Uses strategies and visual models to solve authentic word problems with multiplication using factors less than or equal to 5, including unknowns, using grouping models in situations involving equal groups and arrays.	Uses strategies and visual models to solve authentic word problems with multiplication within 100, including unknowns, using grouping models and equations in situations involving equal groups and arrays.	Uses strategies and visual models to solve authentic word problems with multiplication within 100, including unknowns, using grouping models and equations.	Creates and solves authentic word problems with multiplication within 100, including unknowns, using grouping models and equations, strategies, and visual models.
Range	3.AR.OA.5	Uses strategies and visual models to solve authentic word problems with division using divisors less than or equal to 5, including unknowns, using grouping models in situations involving equal groups and arrays.	Uses strategies and visual models to solve authentic word problems with division within 100, including unknowns, using grouping models and equations in situations involving equal groups and arrays.	Uses strategies and visual models to solve authentic word problems with division within 100, including unknowns, using grouping models and equations.	Creates and solves authentic word problems with division within 100, including unknowns, using grouping models and equations, strategies, and visual models.
Range	3.AR.OA.6	Identifies additive arithmetic patterns using visual supports, such as an addition table.	Identifies multiplicative and subtractive arithmetic patterns using visual supports.	Identifies arithmetic patterns and explains them using properties of operations.	Creates and extends arithmetic patterns, explains patterns using properties of operations.
Geometry and Measurement					
Range	3.GM.G.1	Identifies lines, angles, and parallel lines in two-dimensional shapes.	Identifies lines, angles, and perpendicular and parallel lines in two-dimensional shapes.	Identifies lines, angles, (right, acute, obtuse), and perpendicular and parallel lines in two-dimensional shapes.	Identifies lines, angles, (right, acute, obtuse), and perpendicular and parallel lines in two-dimensional shapes and explains definitions using informal terms.
Range	3.GM.G.2	Recognizes that parallelograms, rectangles, and squares are types of quadrilaterals based on their attributes.	Recognizes that quadrilaterals have 4 sides and sorts parallelograms, rectangles, and squares into categories based on attributes.	Sorts quadrilaterals into categories (quadrilateral, trapezoid, parallelogram, rectangle, rhombus, square) based on attributes.	Sorts any quadrilateral into categories based on attributes and can identify the attributes that define a larger category.
Range	3.GM.G.3	Identifies one or more line of symmetry in rectangles, rhombi, and squares.	Identifies all lines of symmetry in quadrilaterals that are symmetrical.	Identifies all lines of symmetry in quadrilaterals, including identifying when no lines of symmetry are present.	Determines all lines of symmetry in quadrilaterals regardless of their orientations or overall sizes.
Range	3.GM.M.1	Identifies lengths to the nearest inch or half inch when given a ruler marked with halves and fourths of an inch.	Identifies lengths to the nearest half or fourth inch or measures lengths to the nearest inch when given a ruler marked with halves and fourths of an inch.	Measures lengths using rulers marked with halves and fourths of an inch.	Recognizes and identifies an error and justifies the correct solution when measuring lengths to the nearest inch, half inch, or quarter inch using rulers marked with halves, fourths, and eighths of an inch.

MATHEMATICS GRADE 3					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Range	3.GM.M.2	Measures and estimates liquid volumes and masses of objects using standard units (grams, kilograms, or liters) with pictorial models.	Measures and estimates liquid volumes and masses of objects using standard units and solves one-step authentic word problems requiring either subtraction or addition involving masses or volume given in the same units (grams, kilograms, or liters).	Measures and estimates liquid volumes and masses of objects using standard units and solves one-step authentic word problems involving masses or volume given in the same units (grams, kilograms, or liters).	Measures and estimates liquid volumes and masses of objects using standard units and solves one-step authentic word problems involving masses or volume given in the same units (grams, kilograms, or liters) and justifies solutions.
Range	3.GM.M.3	Tells time to the nearest minute.	Tells and writes time to the nearest minute.	Tells and writes time to the nearest minute and measures the number of minutes between two times.	Recognizes and identifies an error and justifies the correct solution when measuring times to the nearest minute.
Range	3.GM.M.4	Solves elapsed-time authentic word problems on the hour with pictorial models.	Solves elapsed-time authentic word problems on the hour using a variety of strategies.	Solves elapsed-time authentic word problems on the hour and the half-hour using a variety of strategies.	Solves one-step authentic word problems involving addition and subtraction of time intervals in minutes including those that cross between A.M. and P.M. times.
Range	3.GM.M.5	Solves authentic word problems involving dollar bills, quarters, dimes, nickels, and pennies with supports, such as pictorial models, decimal notation is not required.	Solves authentic word problems involving dollar bills, quarters, dimes, nickels, and pennies, decimal notation is not required.	Solves authentic word problems involving dollar bills, quarters, dimes, nickels, and pennies using the \$ and ¢ symbols appropriately, decimal notation is not required.	Solves authentic word problems involving dollar bills, quarters, dimes, nickels, and pennies using the \$ and ¢ symbols appropriately and justifies thinking, decimal notation is not required.
Range	3.GM.M.6	Finds the perimeter of rectangles (given the side lengths).	Solves problems involving perimeters of rectangles given the side lengths or when given the perimeter and one unknown side length when a model is provided.	Solves problems involving perimeters of rectangles given the side lengths or when given the perimeter and unknown side length(s).	Solves authentic word problems involving perimeters of rectangles given the side lengths or when given the perimeter and unknown side length(s), including when a model is not provided.
Range	3.GM.M.7	Recognizes area as an attribute of plane figures and understands what a square unit is.	Recognizes area as an attribute of plane figures and understands that area is measured using square units.	Recognizes area as an attribute of plane figures and understands concepts of area measurement.	Recognizes area as an attribute of plane figures and explains concepts of area measurement.
Range	3.GM.M.8	Finds the area of a rectangle with whole-number side lengths by counting unit squares.	Finds the area of a rectangle with whole-number side lengths by modeling with unit squares.	Finds the area of a rectangle with whole-number side lengths by modeling with unit squares and shows that area can be additive and is the same as would be found by multiplying the side lengths.	Finds the area of a rectangle with whole-number side lengths with or without modeling with unit squares and explains that area can be additive and is the same as would be found by multiplying the side lengths.
Data, Probability, and Statistics					
Range	3.DPS.D.1	Collects, organizes, and represents data with four categories using scaled pictures and bar graphs (with a scale factor of 1 or 5) with supports, such as using a model as a guide.	Formulates questions and collects, organizes, and represents data with four categories using scaled pictures and bar graphs, with supports, such as using a model as a guide.	Formulates questions and collects, organizes, and represents data with more than four categories using scaled pictures and bar graphs.	Formulates questions and collects, organizes, and represents data with more than four categories using scaled pictures and bar graphs and explains which visual representation best displays the data.
Range	3.DPS.D.2	Matches line plots and data sets where the horizontal scale is marked whole number or halves.	Matches line plots and data sets where the horizontal scale is marked whole numbers, halves, or fourths.	Uses data to create line plots, where the horizontal scale is marked in whole numbers, halves, or fourths of a unit.	Uses data to create line plots where the horizontal scale is marked in whole numbers, halves, or fourths of a unit and answers questions about the line plot.
Range	3.DPS.D.3	Makes simple statements to solve one-step "how many more" and "how many less" problems using information from the graphs.	Analyzes data and makes simple statements to solve one-step problems using information from the graphs.	Analyzes data and makes simple statements to solve one- and two-step problems using information from the graphs.	Identifies errors and justifies solutions to one- or two-step problems using information from the graphs.

MATHEMATICS GRADE 4					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Policy		The Level 1 student has not yet reached the approaching proficiency level in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student consistently performs below the standard for the grade level/course, is inconsistently able to access grade-level content, and only engages with higher-order thinking skills with extensive support.	The Level 2 student is approaching proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student generally performs slightly below the standard for the grade level/course, is sometimes able to access grade-level content, and engages in higher-order thinking skills with some independence and support.	The Level 3 student is proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student regularly performs at the standard for the grade level/course, is able to access grade-level content, and engages in higher-order thinking skills with independence and minimal support.	The Level 4 student is highly proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student regularly performs independently or significantly above the standard for the grade level/course, is able to access above grade-level content, and engages in higher order-thinking skills independently.
		The Level 1 Student	The Level 2 Student	The Level 3 Student	The Level 4 Student
Number and Operations					
Range	4.NO.CC.1	Reads numbers to the hundred thousands place including word and standard form and writes numbers to the hundred thousands place in standard form.	Reads numbers to the hundred thousands place including word, standard, and expanded form and writes numbers to the hundred thousands place in standard and expanded form.	Reads numbers to the millions place including word, standard, and expanded form and writes numbers to the millions place, including standard and expanded form.	Reads and writes numbers in word, standard, and expanded forms to the millions place in authentic and mathematical contexts in a variety of formats. (e.g., 3461 = 3000+400+60+1, 3400+60+1, 3x1000+4x100+6x10+1)
Range	4.NO.NBT.1	Recognizes that a digit in one place represents 10 times as much as it represents in the place to its right (within 10,000) with visual representations.	Recognizes that a digit in one place represents 10 times as much as it represents in the place to its right (within 100,000).	Recognizes that a digit in one place represents 10 times as much as it represents in the place to its right (for numbers up to and including 1,000,000).	Recognizes the relationship between place values and uses strategies in context to determine the value of any given digit.
Range	4.NO.NBT.2	Compares two numbers up to the hundred thousands place using symbols >, <, and =. Justifies comparisons based on the value of the digits, including using visual models for support.	Compares two numbers up to the millions place and decimals to the hundredths place, using symbols >, <, and =.	Compares two numbers (up to a million) and decimals to the hundredths place, using symbols >, <, and =. Justifies comparisons based on the value of the digits.	Compares two numbers (up to a million) and decimals to the hundredths place in word, standard, or expanded form using symbols and justifies comparisons based on the values of thousands, hundreds, tens, and ones.
Range	4.NO.NBT.3	Applies place value understanding to round multi-digit whole numbers to any place within 10,000.	Applies place value understanding to round multi-digit whole numbers to any place within 100,000.	Applies place value understanding to round multi-digit whole numbers to any place within 1,000,000.	Applies rounding strategies in authentic situations.
Range	4.NO.NBT.4	Adds and subtracts multi-digit whole numbers using the standard algorithm without regrouping.	Adds and subtracts multi-digit whole numbers using the standard algorithm with visual supports.	Adds and subtracts multi-digit whole numbers using the standard algorithm.	Flexibly adds and subtracts multi-digit whole numbers using various strategies, including the standard algorithm, to solve for unknowns in any position.
Range	4.NO.NBT.5	Multiplies a whole number (of up to three digits) by a single-digit whole number with supports.	Multiplies a whole number (of up to four digits) by a single-digit whole number.	Multiplies a whole number (of up to four digits) by a single-digit whole number and multiplies two two-digit numbers and shows and justifies the calculation using equations, rectangular arrays, and/or area models.	Multiplies a whole number (of up to four digits) by a one- or two-digit whole number, identifies an error, and justifies the correct solution.
Range	4.NO.NBT.6	Finds whole-number quotients and remainders (with up to two-digit dividends and single-digit divisors) using place value strategies.	Finds whole-number quotients and remainders (with up to three-digit dividends and single-digit divisors) using place value strategies.	Finds whole-number quotients and remainders (with up to four-digit dividends and single-digit divisors), in context, using place value strategies. Shows and justifies the calculation by using equations, rectangular arrays, and/or area models.	Finds whole-number quotients and remainders (with up to four-digit dividends and single-digit divisors), in context, using place value or other strategies, identifies an error, and justifies the correct solution.
Range	4.NO.NF.1	Expresses a fraction with denominator 10 as an equivalent fraction with denominator 100 and generates decimal notation for fractions with a denominator of 10, with supports.	Expresses a fraction with denominator 10 as an equivalent fraction with denominator 100 and generates decimal notation for fractions with a denominator of 10 or 100, with supports.	Expresses equivalent fractions with a denominator of 10 and a denominator of 100 to generate a decimal notation.	Demonstrates knowledge of decimal notation for fractions with denominators of 10 or 100 by converting a number with decimal notation to a decimal fraction when solving authentic math problems.
Range	4.NO.NF.2	Explains and demonstrates that an improper fraction can be decomposed into a sum of wholes and a proper fraction using visual fraction models and reasoning strategies (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).	Explains and demonstrates that the whole number in a mixed number can be represented by an equivalent fraction with the same denominator as the mixed number using visual fraction models and reasoning strategies (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).	Explains and demonstrates how a mixed number is equivalent to a fraction greater than one and how a fraction greater than one is equal to a mixed number using visual fraction models and reasoning strategies (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).	Applies understanding of mixed numbers and equivalent improper fractions when solving word problems.



MATHEMATICS GRADE 4					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Range	4.NO.NF.3	Identifies equivalent fractions (proper) using denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100 using visual representations.	Generates equivalent fractions (improper and proper) using denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100, using visual representations and number lines.	Generates equivalent fractions (improper and proper) using denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100, using numerical representations, visual representations, and number lines.	Generates equivalent fractions (improper and proper) using denominators of 2, 3, 4, 5, 6, 8,10, 12, and 100 explains why the fractions are equivalent.
Range	4.NO.NF.4	Recognizes that equivalent fractions are generated by multiplying a fraction equivalent to 1 or the properties of multiplication (proper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).	Demonstrates how equivalent fractions are generated by multiplying a fraction equivalent to 1 or the properties of multiplication (proper and improper fractions limited to denominators of 2, 4, 5, 8, 10 and 100).	Demonstrates how equivalent fractions are generated by multiplying a fraction equivalent to 1 or the properties of multiplication (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).	Demonstrates and explains how equivalent fractions are generated by multiplying a fraction equivalent to 1 or the properties of multiplication (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).
Range	4.NO.NF.5	Applies equivalent fraction concepts to compare fractions having different denominators (limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100) and record comparisons using symbols <, >, and = using visual fraction models.	Applies equivalent fraction concepts to compare and order fractions having different denominators (limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100) and record comparisons using symbols <, >, and = and visual fraction models.	Applies equivalent fraction concepts to compare and order fractions having different denominators (limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100) and record comparisons using symbols <, >, and =. Justifies answers using visual fraction models.	Applies equivalent fraction concepts to compare and order fractions having different denominators in real-world math problems. Justifies answers using visual fraction models.
Range	4.NO.NF.6	Solves authentic word problems by adding and subtracting proper fractions with like denominators (limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100) using visual models.	Solves authentic word problems by adding and subtracting proper and improper fractions and mixed numbers with like denominators (limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100) by using visual models.	Solves authentic word problems by adding and subtracting proper and improper fractions and mixed numbers with like denominators (limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).	Solves authentic word problems by adding and subtracting proper and improper fractions and mixed numbers with like denominators and justifies thinking.
Range	4.NO.NF.7	Multiplies unit fractions by whole numbers using visual fraction models (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).	Multiplies proper fractions by whole numbers using visual fraction models (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).	Solves problems by multiplying proper and improper fractions by whole numbers using visual fraction models (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).	Understands and solves more complex word problems by multiplying proper and improper fractions by whole numbers using visual fraction models (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).
Algebraic Reasoning					
Range	4.AR.OA.1	Automatically multiplies and divides through to 6 x 6 and 10s facts that are presented in vertical and horizontal orientations.	Automatically multiplies and divides through to 7 x 7 and 10s facts that are presented in vertical and horizontal orientations.	Automatically multiplies and divides through 10 x 10 when facts are presented in vertical and horizontal orientations.	Automatically multiplies and divides through 10 x 10; analyzes and explains errors and solutions.
Range	4.AR.OA.2	Identifies and applies the properties of operations to solve addition and subtraction equations in vertical or horizontal orientations.	Identifies and applies the properties of operations to solve addition, subtraction, multiplication, and division equations in vertical or horizontal orientations.	Identifies and applies the properties of operations for addition, subtraction, multiplication, and division equations in vertical or horizontal orientations and justifies thinking.	Identifies and applies the properties of operations to solve addition, subtraction, multiplication, and division; analyzes and explains errors and solutions.
Range	4.AR.OA.3	Solves and represents multi-step word problems with equations (without remainders) using the four operations with simple context and scaffolding. The sum, difference, product, or quotient is always the unknown.	Solves and represents multi-step word problems with equations (with or without remainders) using the four operations with simple context and scaffolding. The sum, difference, product, or quotient is always the unknown.	Solves and represents multi-step word problems using the four operations, including interpreting remainders using the four operations and represents problems using equations, including a symbol as an unknown.	Solves complex multi-step word problems with multiple possible solutions and represents them with equations.
Range	4.AR.OA.4	Finds factor pairs within the range of 1 to 20. Determines whether a whole number in the range of 1 to 20 is prime or composite, given visual representations (such as arrays, hundreds chart, and number line).	Finds all factor pairs within the range of 1 to 36 while classifying numbers as prime or composite, given visual representations (such as arrays, hundreds chart, and number line).	Finds factor pairs and multiples within the range of 1-36 while classifying numbers as prime or composite.	Applies the concepts of factors, multiples, and prime and composite numbers in problem-solving contexts within the range of 1-36.
Range	4.AR.OA.5	Matches multiplication equations with multiplicative comparison descriptions.	Recognizes that multiplication equations can be read as a comparison and represents those comparisons as equations using supports.	Interprets multiplication equations as a comparison and represents multiplicative comparisons as multiplication equations.	Recognizes that any two factors and their product can be read as a comparison and uses multiple strategies and creates his or her own to represent and describe those comparisons.
Range	4.AR.OA.6	Generates a number or shape pattern that follows a given rule, using visual models.	Generates a number or shape pattern that follows a given rule.	Generates a number or shape pattern that follows a given rule and identifies apparent features that are not explicit in the rule.	Generates a number or shape pattern that combines two operations for a given rule.

MATHEMATICS GRADE 4					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Geometry and Measurement					
Range	4.GM.G.1	Identifies points, lines, line segments, rays, and lines and classifies angles (right, acute, obtuse).	Identifies and draws points, lines, line segments, rays, and angles (right, acute, obtuse).	Identifies, draws, and labels points, lines, line segments, rays, and angles (right, acute, obtuse).	Creates a two-dimensional shape when given specific attributes.
Range	4.GM.G.2	Identifies and sorts two-dimensional figures based on attributes.	Classifies two-dimensional figures based on the presence or absence of parallel or perpendicular lines.	Classifies two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of specified size.	Constructs two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of specified size.
Range	4.GM.G.3	Identifies lines of symmetry in regular two-dimensional figures.	Draws lines of symmetry for regular two-dimensional figures.	Draws lines of symmetry for regular and irregular two-dimensional figures.	Constructs a figure with a given number of lines of symmetry.
Range	4.GM.M.1	Knows relative size of measurement units within one system of units.	Knows relative size of measurement units within one system of units. Record measurement equivalents in a two-column table, using supports and adjacent units.	Knows the relative sizes of measurement units within one system of units, and records that data in a two-column table.	Determines the appropriate unit needed and expresses the measurement to the level of accuracy needed for a given context.
Range	4.GM.M.2	Generate simple conversions from a larger unit to a smaller unit within a single system of measurement, using the metric system with supports.	Generate simple conversions from a larger unit to a smaller unit within a single system of measurement, both customary and metric systems with supports.	Generate simple conversions from a larger unit to a smaller unit to solve authentic problems within a single system of measurement, both customary and metric systems.	Generate simple conversions from a larger unit to a smaller unit to solve authentic problems and justify answers within a single system of measurement, both customary and metric systems.
Range	4.GM.M.3	Identifies and uses the appropriate tools, operations, and units of metric measurement to solve problems involving time, length, weight, mass, and capacity, using supports.	Identifies and uses the appropriate tools, operations, and units of measurement, both customary and metric, to solve problems involving time, length, weight, mass, and capacity, using supports.	Identifies and uses the appropriate tools, operations, and units of measurement, both customary and metric, to solve problems involving time, length, weight, mass, and capacity.	Identifies and uses the appropriate tools, operations, and units of measurement, both customary and metric, to solve multi-step problems involving time, length, weight, mass, and capacity.
Range	4.GM.M.4	Identifies values for word problems involving dollar bills, quarters, dimes, nickels, and pennies using \$ and ¢ symbols and decimal notation appropriately with the support of pictorial models.	Solves authentic word problems involving dollar bills, quarters, dimes, nickels, and pennies using \$ and ¢ symbols and decimal notation appropriately with the support of pictorial models.	Solves authentic word problems involving dollar bills, quarters, dimes, nickels, and pennies using \$ and ¢ symbols and decimal notation appropriately.	Solves authentic word problems involving dollar bills, quarters, dimes, nickels, and pennies using \$ and ¢ symbols and decimal notation appropriately and justifies thinking.
Range	4.GM.M.5	Applies the area and perimeter formulas for rectangles when given all side measurements, using pictorial models.	Applies the area and perimeter formulas for rectangles, including connected rectangular figures, using pictorial models.	Applies the area and perimeter formulas for rectangles, including connected rectangular figures in problems.	Applies the area and perimeter formulas for rectangles, including connected rectangular figures in authentic and mathematical problems.
Range	4.GM.M.6	Identifies angle measures in whole-number degrees using a protractor in a pictorial model.	Measures angles in whole-number degrees using a protractor.	Measures angles in whole-number degrees using a protractor and draws angles of a specified measure.	Recognizes and identifies an error and justifies the correct solution when measuring angles in whole-number degrees using a protractor and draws angles of a specified measure.
Range	4.GMM.7	Recognizes that angle measure is additive. Solves addition problems to find unknown angles on a diagram with no more than two angles within a 90- degree angle.	Recognizes that angle measure is additive. Solves addition and subtraction problems to find unknown angles on a diagram with no more than two angles within a 180-degree angle.	Recognizes that angle measure is additive. Solves addition and subtraction to find unknown angles on a diagram.	Given angle parameters, decomposes into multiple angles and gives the measure of each angle in relationship to the whole. Students solve authentic mathematical problems to find unknown angles on a diagram.
Data, Probability, and Statistics					
Range	4.DPS.D.1	Collects, organizes, and represent data to reason with math and across disciplines with guided support or models.	Formulate questions and collects, organizes, and represent data to reason with math and across disciplines with guided support or models.	Formulate questions to collect, organize, and represent data to reason with math and across disciplines.	Formulate questions to collect, organize, and explains which visual representation best displays the data when reasoning about math and other disciplines.
Range	4.DPS.D.2	Matches line plots and data sets of measurement in fractions of a unit (with like denominators of 2 or 4).	Matches line plots and data sets of measurement in fractions of a unit (with like denominators of 2 or 4) and uses addition and subtraction of fractions to solve problems involving information in the line plot.	Uses data to create a line plot to display a data set of measurements in fractions of a unit (with like denominators limited to 2, 4, and 8) and uses addition and subtraction of fractions to solve problems involving information in the line plot.	Uses data in a line plot to solve a multi-step word problem.

MATHEMATICS GRADE 4					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Range	4.DPS.D.3	Utilize graphs and diagrams to represent and solve authentic word problems using the four operations involving whole numbers.	Utilize graphs and diagrams to represent and solve authentic word problems using the four operations involving whole numbers and decimals.	Utilize graphs and diagrams to represent and solve authentic word problems using the four operations involving whole numbers, benchmark fractions, and decimals.	Utilize graphs and diagrams to represent and solve authentic multi-step word problems using the four operations involving whole numbers, benchmark fractions, and decimals.

MATHEMATICS GRADE 5					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Policy		The Level 1 student has not yet reached the approaching proficiency level in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student consistently performs below the standard for the grade level/course, is inconsistently able to access grade-level content, and only engages with higher-order thinking skills with extensive support.	The Level 2 student is approaching proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student generally performs slightly below the standard for the grade level/course, is sometimes able to access grade-level content, and engages in higher-order thinking skills with some independence and support.	The Level 3 student is proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student regularly performs at the standard for the grade level/course, is able to access grade-level content, and engages in higher-order thinking skills with independence and minimal support.	The Level 4 student is highly proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student regularly performs independently or significantly above the standard for the grade level/course, is able to access above grade-level content, and engages in higher order-thinking skills independently.
		The Level 1 Student	The Level 2 Student	The Level 3 Student	The Level 4 Student
Number and Operations					
	5.NO.CC.1	Reads decimals to the thousandths place.	Reads and writes decimals to the thousandths place, using standard and word forms.	Reads and writes decimals to the thousandths place, using standard, word, and expanded forms.	Writes numbers in expanded form in a variety of formats (e.g., $347.392 = 7 \times 1 + 3.4 \times 100 + 3 \times (1/10) + 2 \times (1/1000) + (1/100) \times 9$ ).
Range	5.NO.NBT.1	Uses visual models or calculation (in any multi-digit whole number) to recognize a digit in one place represents 10 times as much as it represents in the place to its right, or 1/10 of what it represents in the place to its left.	Uses visual models or calculation (in any multi-digit whole number) to demonstrate that a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	Recognizes (in any multi-digit number, including decimals to thousandths) that a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	Recognizes (in any multi-digit number, including decimals to thousandths) that a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left, in authentic or mathematical context problems.
Range	5.NO.NBT.2	Compares two decimals to the tenths place, using $>$ , $=$ , and $<$ symbols. Justify comparisons based on the value of the digits.	Compares two decimals to the hundredths place, using $>$ , $=$ , and $<$ symbols. Justifies comparisons based on the value of the digits.	Compares two decimals to the thousandths place (with varying place values), using $>$ , $=$ , and $<$ symbols and justifies comparisons based on the value of the digits.	Compares decimals to the thousandths place (with varying place values) and justifies comparisons based on the value of the digits.
Range	5.NO.NBT.3	Applies place value understanding to round numbers including decimal values to the tenths.	Applies place value understanding to round numbers including decimal place values to the tenths or hundredths place.	Applies place value understanding to round decimals to any place.	Applies rounding strategies to round decimals to any place in authentic situations.
Range	5.NO.NBT.4	Multiplies two two-digit numbers presented in vertical or horizontal orientations using strategies, including a standard algorithm.	Multiplies three-digit by two-digit whole numbers presented in vertical or horizontal orientations using strategies, including a standard algorithm.	Multiplies multi-digit whole numbers presented in vertical or horizontal orientations flexibly using strategies, including a standard algorithm.	Multiplies multi-digit whole numbers in vertical or horizontal orientations, in authentic and mathematical contexts, flexibly using strategies, including a standard algorithm.
Range	5.NO.NBT.5	Adds, subtracts, and multiplies decimals to the tenths place, using concrete models and drawings.	Adds, subtracts, and multiplies decimals to the tenths place, using concrete models, drawings, place value strategies, properties of operations and/or relationships.	Adds, subtracts, and multiplies decimals to the hundredths place, using concrete models, drawings, place value strategies, properties of operations and/or relationships.	Adds, subtracts, and multiplies decimals to the hundredths place, using multiple strategies, in a authentic or mathematical context.
Range	5.NO.NBT.6	Finds whole-number quotients and remainders (with up to two-digit dividends and two-digit divisors), using place value strategies.	Finds whole-number quotients and remainders (with up to three-digit dividends and two-digit divisors), using place value strategies.	Finds whole-number quotients and remainders (with up to four digit dividends and two-digit divisors), using place value strategies. Shows and justifies the calculation by using equations, rectangular arrays, and/or area models.	Finds whole-number quotients and remainders (with up to four-digit dividends and two-digit divisors) in context.
Range	5.NO.NBT.7	Continues a given pattern that shows the number of zeroes of the product when multiplying a number by powers of 10.	Recognizes patterns in the number of zeroes of products when multiplying a number by powers of 10. Can use whole number exponents greater than zero to denote powers of 10.	Explains patterns in the number of zeroes of the product when multiplying a number by powers of 10, and explains patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Uses whole-number exponents to denote powers of 10, including 10 to the power of zero.	Interprets a multiplication problem to identify the power of 10 by which one number is greater or less than another.
Range	5.NO.NF.1	Generates equivalent forms of commonly used fractions and decimals (e.g., halves).	Generates equivalent forms of commonly used fractions and decimals (e.g., halves, fourths).	Generates equivalent forms of commonly used fractions and decimals (e.g., halves, fourths, fifths, tenths).	Generates equivalent forms of commonly used fractions and decimals in a word problem context (e.g., halves, fourths, fifths, tenths).
Range	5.NO.NF.2	Explains why multiplying a given number by a fraction less than one results in a product smaller than the given number, using visual models.	Explains why multiplying a given number by a fraction greater than one results in a product greater than the given number and explains why multiplying a given number by a fraction less than one results in a product smaller than the given number, using visual models.	Explains why multiplying a given number by a fraction greater than one results in a product greater than the given number and explains why multiplying a given number by a fraction less than one results in a product smaller than the given number.	Explains why multiplying a given number by a fraction greater than one results in a product greater than the given number and explains why multiplying a given number by a fraction less than one results in a product smaller than the given number within a word problem context.



MATHEMATICS GRADE 5					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Range	5.NO.NF.3	Solves authentic word problems by adding and subtracting fractions with unlike denominators, where one denominator is a multiple of the other denominator, using visual fraction models (excluding mixed numbers and regrouping).	Solves authentic word problems by adding and subtracting fractions and mixed numbers with unlike denominators, where one denominator is a multiple of the other denominator (excluding regrouping), using visual fraction models.	Solves authentic word problems by adding and subtracting fractions and mixed numbers with unlike denominators using a visual fraction model and equations.	Solves authentic word problems by adding and subtracting at least 3 or more fractions or mixed numbers with unlike denominators by replacing given fractions with equivalent fractions.
Range	5.NO.NF.4	Solves authentic word problems by multiplying fractions using visual fraction models or equations to represent the problem (limited to fractions with single-digit numerators or denominators).	Solves authentic word problems by multiplying fractions using visual fraction models or equations to represent the problem.	Solves authentic word problems by multiplying fractions and mixed numbers, using visual fraction models and equations.	Solve multi-step authentic word problems by multiplying fraction and mixed numbers that could include multi-digit numerators or denominators.

MATHEMATICS GRADE 5					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Algebraic Reasoning					
Range	5.AR.OA.1	Automatically multiply through 11 x 11 when presented facts in a horizontal or vertical orientation.	Automatically multiply and divide through 11 x 11 when presented facts in a horizontal or vertical orientation.	Automatically multiply and divide through 12 x 12 when presented facts in a horizontal or vertical orientation.	Automatically multiply and divide through 12 x 12; analyzes and explains errors and solutions.
Range	5.AR.OA.2	Analyzes problems using the order of operations to solve and evaluate expressions using parentheses (without nesting).	Analyzes problems using the order of operations to solve and evaluate expressions using parentheses and brackets.	Analyzes problems using the order of operations to solve and evaluate expressions including parentheses, brackets, and/or braces; justifies thinking.	Analyzes problems using the order of operations to solve and inserts parentheses (without nesting), in equations to make a statement true.
Range	5.AR.OA.3	Writes a numerical expression, using one operation, from a written statement (e.g., divide 144 by 12).	Writes simple numerical expressions (limited to inverse operations) and interprets numerical expressions without evaluating them.	Writes simple numerical expressions (limited to two operations; e.g., "divide 144 by 12, and then subtract 9") and interprets numerical expressions without evaluating them.	Writes numerical expressions using multiple operations, involving real-world and mathematical contexts.
Range	5.AR.OA.4	Finds factor pairs within the range of 1 to 50. Determines whether a whole number in the range of 1 to 50 is prime or composite, given visual representations (such as arrays, hundreds chart, and number line).	Finds all factor pairs for numbers in the range of 1 to 100. Determines whether a whole number in the range of 1 to 75 is prime or composite, given visual representations (such as arrays, hundreds chart, number line).	Finds factor pairs and multiples within the range of 1-100 while classifying numbers as prime or composite.	Applies the concepts of factors, multiples and prime and composite numbers in problem-solving contexts within the range of 1-100.
Range	5.AR.OA.5	Generates or continues two numerical patterns (when given a table), using two given rules.	Generates or continues two numerical patterns using two given rules.	Generates two numerical patterns using two given rules and forms ordered pairs consisting of corresponding terms from the two patterns. (Graphing on a coordinate plane.)	Generates two numerical patterns using two multi-step given rules, in mathematical contexts. Explains the relationship between corresponding terms.
Geometry and Measurement					
Range	5.GM.G.1	Recalls the defining characteristics of two-dimensional (fifth grade) figures based on properties limited to sides and angles.	Identifies two-dimensional (fifth grade) figures based on their properties (sides and angles).	Classifies two-dimensional figures in a hierarchy based on properties.	Draws or constructs specific two-dimensional figures according to the definitions provided, attributes described, or categories given.
Range	5.GM.G.2	Identifies the x-coordinate and y-coordinate of a point in the first quadrant of the coordinate plane.	Identifies the x-coordinate and y-coordinate to name points in the first quadrant of the coordinate plane.	Identifies the x-coordinate and y-coordinate to graph and name points in the first quadrant of the coordinate plane.	Identifies the x-coordinate and y-coordinate to graph and name more than one point in the first quadrant of the coordinate plane. Graphs and uses
Range	5.GM.G.3	Identifies the key components of the coordinate plane (x - axis, x - coordinate, y -axis, y -coordinate, and origin) and graphs given points in the first quadrant of the coordinate plane.	Interprets authentic word problems and forms ordered pairs in the first quadrant of the coordinate plane.	Solves authentic word problems by forming ordered pairs and graphing points in the first quadrant of the coordinate plane.	Using authentic data, creates a representation using ordered pairs and point in the coordinate plane and draws conclusions based on the data presented.
Range	5.GM.M.1	Converts among different-sized standard measurement units within a given measurement system.	Converts among different-sized standard measurement units within a given measurement system; uses these conversions to solve single- step problems, using manipulatives or visual models.	Converts among different-sized standard measurement units within a given measurement system; uses these conversions in solving multi-step, authentic problems.	Creates authentic, multi-step problems. Chooses the appropriate measurement unit based on the given context.
Range	5.GM.M.2	Finds the area of a rectangle (including connected rectangular figure with fractional side lengths) by showing the product of a fraction by a whole number using repeated addition or visual fraction models. Finds the perimeter of a rectangle (including rectangular figure with fractional side lengths) by using visual fraction models.	Finds the area of a rectangle (including connected rectangular figure with fractional side lengths) by showing the product of two fractions using an area model. Finds the perimeter of a rectangle (including rectangular figure with fractional side lengths)	Find the area and perimeter of a rectangle, including connected rectangular figure with fractional side lengths.	Demonstrates reasoning about fractions in both an additive and multiplicative sense with different wholes, and displays the quantities with visual models.
Range	5.GM.M.3	Identifies that unit cubes are used to measure volume in rectangular prisms.	Recognizes volume as an attribute of rectangular prisms and measures volume by identifying a strategy to count unit cubes.	Recognizes volume as an attribute of rectangular prisms and measures volume by counting unit cubes with or without all cubes being visible.	Compares the volumes of different prisms by using unit cubes with or without all cubes being visible. Constructs rectangular prisms that have a volume that is greater than, less than, or equal to the volume of another
Data, Probability, and Statistics					
Range	5.DPS.D.1	Identify a line plot that matches a data set of measurements in fractions of a unit (1/2, 1/4, 1/8), and uses addition and subtraction to solve problems involving information presented in line plots.	Identify a line plot that matches a data set of measurements in fractions of a unit (1/2, 1/4, 1/8) and uses grade-level operations for fractions with common denominators to solve problems involving information presented in line plots.	Uses data to create line plots to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8) and uses grade-level operations for fractions to solve problems involving information presented in line plots.	Uses data to create line plots to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8), solves multi-step word problems using the four operations, and interprets or justifies the solution.
Range	5.DPS.D.2	Selects appropriate graph and represents various data points, including whole numbers, fractions, and decimals.	Utilizes graphs and diagrams to represent, analyze, and solve authentic problems using information presented in one table or line plot, including whole numbers, fractions, and decimals.	Utilizes graphs and diagrams to represent, analyze, and solve authentic problems using information presented in one or more tables or line plots, including whole numbers, fractions, and decimals.	Utilizes graphs and diagrams to represent, analyze, and solve authentic problems using information presented in one or more tables or line plots, including whole numbers, fractions, and decimals. Justifies and explains thinking.

MATHEMATICS GRADE 6					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Policy		The Level 1 student has not yet reached the approaching proficiency level in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student consistently performs below the standard for the grade level/course, is inconsistently able to access grade-level content, and only engages with higher-order thinking skills with extensive support.	The Level 2 student is approaching proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student generally performs slightly below the standard for the grade level/course, is sometimes able to access grade-level content, and engages in higher-order thinking skills with some independence and support.	The Level 3 student is proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student regularly performs at the standard for the grade level/course, is able to access grade-level content, and engages in higher-order thinking skills with independence and minimal support.	The Level 4 student is highly proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student regularly performs independently or significantly above the standard for the grade level/course, is able to access above grade-level content, and engages in higher order-thinking skills independently.
		The Level 1 Student	The Level 2 Student	The Level 3 Student	The Level 4 Student
Number and Operations					
Range	6.NO.NS.1	Plots integers on the horizontal or vertical number line (with whole-number increments), extending the counting pattern to integers.	Plots integers on the horizontal or vertical number line. In a given situation (e.g., elevation, sea level), student is able to determine the meaning of zero.	Demonstrates that positive and negative numbers are used together to describe quantities having opposite directions, values, and equal distances from 0 (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); uses positive and negative numbers to represent quantities in authentic contexts, explaining the meaning of 0 in each situation. (May use any rational number, including fractions and decimals.)	Recognizes patterns and makes generalizations about characteristics of positive and negative numbers. (May use any rational number, including fractions and decimals.)
Range	6.NO.NS.2	Compares two rational numbers on a number line diagram. Writes the comparison using mathematical notation.	Determines the greater or lesser rational number, in a real-world context. Uses mathematical notation and words to express these statements of order.	Writes, interprets, and explains statements of order for rational numbers on a number line diagram and in authentic contexts.	Draws conclusions about a real-world situation involving rational numbers and compares values.
Range	6.NO.O.1	Finds whole-number quotients (with up to four-digit dividends and one-digit divisors).	Finds quotients of whole numbers (with up to four-digit dividends and two-digit divisors).	Divides multi-digit numbers up to four-digit dividends and two-digit divisors using strategies or procedures. Illustrates and explains the calculation by using equations, rectangular arrays, and/or area models.	Compare two or more division strategies and assess the efficiency of the methods.
Range	6.NO.O.2	Adds or subtracts multi-digit decimals up to the hundredths place or fractions using strategies based on place value, the properties of operations, and/or the relationship between operations.	Add and subtracts multi-digit decimals up to the hundredths place or fractions, using strategies based on place value, the properties of operations, and/or the relationship between operations	Adds and subtracts, multi-digit decimals up to the hundredths place and fractions, including authentic problems.	Compares two or more strategies to add and subtract multi-digit decimals up to the hundredths place and fractions.
Range	6.NO.O.3	Applies multiplication or division of fractions or decimals to solve problems using visual models.	Applies multiplication and division of fractions and decimals to solve and interpret problems using visual models.	Applies multiplication and division of fractions and decimals to solve and interpret problems using visual models, including authentic problems.	Applies multiplication and division of fractions and decimals to solve and interpret problems using visual models, including authentic problems, and justifies solutions.
Range	6.NO.O.4	Finds common factors (less than or equal to 50) and common multiples (less than or equal to 10), using a visual model or strategies.	Finds the greatest common factor of two whole numbers (less than or equal to 50) and the least common multiple of two whole numbers (less than or equal to 10).	Finds the greatest common factor of two whole numbers (less than or equal to 100) and the least common multiple of two whole numbers (less than or equal to 12).	Interprets a context to construct an equivalent expression, using greatest common factors and least common multiples.
Algebraic Reasoning					
Range	6.AR.RP.1	Identifies the ratio relationship of a visual model.	Describes the concept of ratio relationship between two quantities using basic ratio language.	Describes the concept of a ratio relationship between two quantities using ratio language and visual models.	Uses and connects between representations for ratio situations. For example, 7 blue marbles and 8 red marbles (e.g., 7:8, 7/8, 8:7, 7 to 8, 8/15, 8 red marbles to 15 total marbles).
Range	6.AR.RP.2	Identifies a unit rate when given a ratio relationship between two quantities using visual models.	Calculates a unit rate when given a ratio relationship between two quantities using rate language and visual models.	Describes and calculates a unit rate when given a ratio relationship between two quantities using rate language and visual models.	Calculates a unit rate with multiple steps. (e.g., A car travels 125 miles and uses 5 gallons of gasoline. Ten gallons of gas cost \$30. What is the cost of gasoline for each mile the car travels?)
Range	6.AR.RP.3	Identifies equivalent ratios.	Makes tables of equivalent ratios, tape diagrams, double number line diagrams, and equations.	Makes and uses tables of equivalent ratios, tape diagrams, double number line diagrams, and equations to solve problems involving ratios, rates, and unit rates, including authentic problems.	Creates and solves authentic problems using ratio and rate reasoning.

MATHEMATICS GRADE 6					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Range	6.AR.RP.4	Recognizes the meaning of percent of a quantity as a rate per hundred.	Calculates the percent of a quantity.	Calculates the percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); Solves problems using ratio reasoning involving finding the whole, given a part and the percent.	Solves non-routine authentic or numerical problems involving percent.
Range	6.AR.RP.5	Converts measurement units within one measurement system using ratio reasoning given conversion factors in a table.	Converts measurement units within one measurement system using ratio reasoning given conversion factors in different representations.	Converts measurement units within and between measurement systems using ratio reasoning given conversion factors.	Applies ratio reasoning to authentic word problems where students convert measurement units.
Range	6.AR.EE.1	Recognizes exponential notation as repeated multiplication (e.g., $2 \times 2 \times 2 = 2^3$ ).	Writes and evaluates a single term in numerical expressions involving whole-number exponents (e.g., $7^2 = 49$ or $49 = 7^2$ ).	Reads, writes, and evaluates numerical expressions, including expressions with whole number exponents and grouping symbols.	Reads, writes, and evaluates numerical expressions involving whole-number exponents and grouping symbols in authentic contexts.
Range	6.AR.EE.2	Identifies an algebraic expression that represents simple and authentic situations (with one variable).	Reads and evaluates algebraic expressions, including expressions with whole number exponents and recognizes one or more parts of an expression as single entities.	Reads and evaluates algebraic expressions, including expressions with whole number exponents and grouping symbols. Writes algebraic expressions to represent simple and authentic situations.	Reads and evaluates algebraic expressions, including expressions with whole number exponents and grouping symbols, writes algebraic expressions to represent simple and authentic situations, and justifies thinking.
Range	6.AR.EE.3	Identifies when two expressions are equivalent.	Applies a single property of operations to generate equivalent expressions.	Identifies when two expressions are equivalent. Applies the properties of operations to generate equivalent expressions.	Uses an authentic context to construct multiple equivalent expressions.
Range	6.AR.EE.4	Determines whether a given number is a solution to an equation(with a single operation)-	Determines whether a given number is a solution to an equation or inequality (with a single operation).	Describes the concept of a solution to an equation or an inequality and determines whether a given number is a solution to an equation or an inequality.	Determines a set of values that makes an equation or inequality true.
Range	6.AR.EE.5	Solves $x + p = q$ and $px = q$ (for cases in which p and q are whole numbers) with a visual/manipulative model.	Solves $x + p = q$ and $px = q$ (for cases in which p and q are non-negative whole numbers or decimals).	Writes and solves equations of the form $x + p = q$ and $px = q$ for cases in which p and q are non- negative whole numbers or decimals, including authentic problems.	Creates an authentic problem that can be solved with a given equation.
Range	6.AR.EE.6	Identifies solutions of inequalities of the form $x > c$ or $x < c$ on a number line diagram.	Given a number line diagram, writes an inequality of the form $x > c$ or $x < c$ ; or, given an inequality of the form $x > c$ or $x < c$ , graphs solutions on a number line diagram.	Writes a statement of inequality of the form $x > c$ or $x < c$ to represent a constraint or condition and recognizes that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions, and represents solutions of such inequalities on number line diagrams.	Writes an authentic problem to represent a constraint given an inequality of the form $x > c$ or $x < c$ .
Geometry and Measurement					
Range	6.GM.AV.1	Calculates the areas of rectangles.	Calculates the areas of triangles and quadrilaterals by composing and/or decomposing them into rectangles and triangles given all the measurements.	Derives the relationship of the areas of triangles using the area of rectangles. Calculates the areas of triangles and quadrilaterals by composing and/or decomposing them into rectangles and triangles, including authentic problems.	Solves authentic problems involving the areas of triangles and quadrilaterals including decimal and fractional measurements and justifies solutions.
Range	6.GM.AV.2	Recognizes the concept of volume of a right rectangular prism. Applies given formulas to calculate the volume of a right rectangular prism with whole- number edge lengths.	Describes the concept of volume of a right rectangular prism. Applies given formulas to calculate the volume of a right rectangular prism with one fractional edge length.	Describes the concept of volume of a right rectangular prism. Applies given formulas to calculate the volume of right rectangular prisms, including fractional edge lengths, including authentic problems.	Given the volume of a right rectangular prism with fractional edge lengths, finds the missing fractional edge length in authentic problems.
Range	6.GM.GF.1	Identifies or positions ordered pairs of rational numbers in the first quadrant of a coordinate plane.	Identifies and positions ordered pairs of integers in all four quadrants of a coordinate plane.	Identifies and positions ordered pairs of rational numbers in all four quadrants of a coordinate plane.	Identifies and positions ordered pairs of rational numbers in all four quadrants of a coordinate plane to solve real-world, multi- step problems.
Range	6.GM.GF.2	Determines the length of a side joining points with the same first coordinate or the same second coordinate.	Draws polygons in the coordinate plane given coordinates for the vertices and determines the length of a side joining points with the same first or second coordinate.	Draws polygons in the coordinate plane given coordinates for the vertices. Determines the length of a side joining points with the same first or second coordinate, including authentic problems.	Finds the missing vertex of a regular polygon when given the other vertices in the coordinate plane in an authentic problem.



MATHEMATICS GRADE 6					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Range	6.GM.GF.3	Represents three-dimensional figures using nets made up of rectangles and/or triangles.	Calculates the surface area of prisms with rectangular base using nets.	Represents three-dimensional figures using nets made up of rectangles and triangles. Calculates the surface area of prisms with rectangular and triangular bases using nets, including authentic problems.	Calculates the surface area of prisms with rectangular and triangular bases using nets, including authentic problems with fractional and decimal measurements.
Data, Probability, and Statistics					
Range	6.DPS.D.1	Recognizes a statistical question from a list of questions.	Writes a statistical question.	Writes a statistical question that can be answered using measures of the center or variability of a data set.	Writes a statistical question that can be answered using measures of the center or variability of a data set given a context.
Range	6.DPS.D.2	Recognizes measures of center (median and mean) and variability (range and mean absolute deviation).	Recognizes and calculates measures of center- (mean, median, and/or mode) and variability (range and mean absolute deviation).	Calculates measures of center (median and mean) and variability (range and mean absolute deviation) to answer a statistical question. Identifies mode(s) if they exist.	Describes the affect of additional data points on the measures of center and variability in a numerical data set.
Range	6.DPS.D.3	Identifies outliers by observation.	Identifies outliers by observation and describes their effect on measures of center and variability.	Identifies outliers by observation and describes their effect on measures of center and variability. Justifies which measures would be appropriate to answer a statistical question.	Analyzes measures of center and variability to answer a statistical question when given a data set.
Range	6.DPS.D.4	Identifies an appropriate display of numerical data in plots on a number line, including dot plots or histograms.	Displays numerical data on a number line, including dot plots and describes any overall patterns in data, such as gaps and clusters.	Displays numerical data in plots on a number line, including dot plots and histogram and describes any overall patterns in data, such as gaps, clusters, and skews.	Constructs a histogram from data displayed in a dot plot and creates dot plots and histograms based on given patterns in data

MATHEMATICS GRADE 7					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Policy		The Level 1 student has not yet reached the approaching proficiency level in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student consistently performs below the standard for the grade level/course, is-inconsistently able to access grade-level content, and only engages with higher-order thinking skills with extensive support.	The Level 2 student is approaching proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student generally performs slightly below the standard for the grade level/course, is sometimes able to access grade-level content, and engages in higher-order thinking skills with some independence and support.	The Level 3 student is proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student regularly performs at the standard for the grade level/course, is able to access grade-level content, and engages in higher-order thinking skills with independence and minimal support.	The Level 4 student is highly proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student regularly performs independently or significantly above the standard for the grade level/course, is able to access above grade-level content, and engages in higher order-thinking skills independently.
		<b>The Level 1 Student</b>	<b>The Level 2 Student</b>	<b>The Level 3 Student</b>	<b>The Level 4 Student</b>
<b>Number and Operations</b>					
Range	7.NO.NS.1	Determines the absolute value of an integer presented using the absolute value notation.	Determines the absolute value of a rational number presented using the absolute value notation.	Describes the absolute value of a number as its distance from zero on a number line.	Determines the number(s) that have a given absolute value.
Range	7.NO.NS.2	Determines equivalent fractions for terminating decimal values.	Determines equivalent decimal values for fractions up to a denominator of 10.	Recognizes common fractions and decimal equivalencies up to a denominator of 10 and converts a rational number to a decimal using technology.	Recognizes common fractions and decimal equivalencies up to a denominator of 100 and converts a decimal to a rational number using technology.
Range	7.NO.O.1	Adds, subtracts, multiplies, or divides integers using visual models in one-step problems.	Adds, subtracts, multiplies, and divides integers using visual models and properties of operations in one-step problems.	Adds, subtracts, multiplies, and divides integers using visual models and properties of operations in multi-step problems, including authentic problems.	Justifies the steps taken to solve a multi-step problem, including authentic problems, which involve addition, subtraction, multiplication and division of rational numbers.
Range	7.NO.O.2	Adds, subtracts, multiplies, or divides non-negative fractions in one-step problems.	Adds, subtracts, multiplies, and divides non-negative fractions in one-step problems.	Adds, subtracts, multiplies, and divides non-negative fractions in multi-step problems, including authentic problems.	Justifies the steps taken when adding, subtracting, multiplying, and dividing non-negative fractions in multi-step problems, including authentic problems.
Range	7.NO.O.3	Adds, subtracts, multiplies, or divides non-negative decimals to the hundredth place in one-step problems using strategies or procedures.	Add, subtracts, multiplies, and divides non-negative decimals to the hundredth place in one-step problems using strategies or procedures.	Adds, subtracts, multiplies, and divides non-negative decimals to the hundredth place in multi-step problems using strategies or procedures, including authentic problems.	Justifies the steps taken to solve a multi-step problem, including authentic problems, which involve addition, subtraction, multiplication, and division of non-negative decimals to the hundredth place.
<b>Algebraic Reasoning</b>					
Range	7.AR.RP.1	Calculates unit rates associated with ratios of whole numbers having like units.	Calculates unit rates associated with ratios of fractions, including lengths with like or different units.	Calculate unit rates associated with ratios of rational numbers, including ratios of lengths, areas, and other quantities measured in like or different units.	Calculates unit rates associated with ratios of two mixed numbers having like or different units.
Range	7.AR.RP.2	Decides whether dependent and independent variables are in a proportional relationship in a representation that includes (0, 0).	Decides whether dependent and independent variables are in a proportional relationship using graphs and tables and explains what any point (x, y) on the graph of a proportional relationship means in terms of the situation, and can identify the unit rate when given the point (1, k).	Analyzes the relationship between the dependent and independent variables of a proportional relationship using graphs and tables and explains what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0,0) and (1, k) where k is the unit rate.	Creates a different representation (graph, table) between dependent and independent variables that would represent the same proportional relationship when given one representation (graph, table) and identifies a point (x, y) on the same graph as the point (1, k) for a proportional relationship and interprets the meaning of (x, y) in terms of the situation.
Range	7.AR.RP.3	Identifies the constant of proportionality in tables, graphs, equations, diagrams, and descriptions of proportional relationships in a representation that includes (1, k) and identifies the equation for a proportional relationship of the form $y = kx$ , where k is the constant of proportionality.	Identifies the constant of proportionality in tables, graphs, equations, diagrams, and-descriptions of proportional relationships and represents proportional relationships by an equation of the form $y = kx$ , where k is the constant of proportionality.	Identifies the constant of proportionality in tables, graphs, equations, diagrams, and descriptions of proportional relationships represents proportional relationships by an equation of the form $y = kx$ , where k is the constant of proportionality, and describes the meaning of each variable (y, k ,x) in the context of the situation.	Models a constant of proportionality using tables, graphs, equations, diagrams, and descriptions of proportional relationships and represents proportional relationships by an equation of the form $y = kx$ , where k is the constant of proportionality, and describes the meaning of each variable (y, k ,x) in the context of the situation.
Range	7.AR.RP.4	Uses proportional relationships to solve simple problems involving ratios and percents and finds actual lengths given a geometric figure and a scale factor.	Uses proportional relationships to solve simple problems involving ratios, percents, and scale drawings of geometric figures.	Uses proportional relationships to solve multi-step problems involving ratios, percents, and scale drawings of geometric figures, including authentic problems.	Explains the proportional relationship -given in a ratio, percent, or scale drawing of geometric figures, including authentic problems, and creates equivalent proportional relationships.

MATHEMATICS GRADE 7					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Range	7.AR.EE.1	Applies properties of operations used to add, subtract, factor, and expand linear expressions involving variables and integers; identifies equivalent expressions.	Applies-properties of operations as strategies to add, subtract, factor, and expand linear expressions involving variables and integers, with an emphasis on writing equivalent expressions.	Applies properties of operations as strategies to add, subtract, factor, and expand linear expressions involving variables, integers, and/or non-negative fractions and decimals with an emphasis on writing equivalent expressions.	Applies and justifies properties of operations as strategies to add, subtract, factor, and expand linear expressions involving variables, integers, and/or non-negative fractions and decimals and creates equivalent expressions and justifies the reason for equivalency.
Range	7.AR.EE.2	Solves equations of the form $px + q = r$ and $p(x + q) = r$ when p, q, and r are whole numbers.	Solves equations of the form $px + q = r$ and $p(x + q) = r$ when q, and r are integers and p is a whole number.	Writes and solves equations of the form $px + q = r$ and $p(x + q) = r$ when q and r are integers and p is an integer or positive fraction/decimal, including authentic problems.	Creates an authentic context to model equations of the form $px + q = r$ and $p(x + q) = r$ , and explains what the solution means.
Range	7.AR.EE.3	Solves simple inequalities where coefficients and solutions are integers and identifies the graph for a solution set of the inequality.	Solves simple inequalities where coefficients and solutions are integers and/or non-negative fractions and decimals, including authentic problems and graphs the solution set of the inequality.	Writes and solves one- or two-step inequalities where coefficients and solutions are integers and/or non-negative fractions and decimals, including authentic problems and graphs the solution set of the inequality and interprets the context of the problem.	Solves and justifies one- or two-step inequalities where coefficients and solutions are integers and/or non-negative fractions and decimals, including authentic problems and explains the graph of a solution set of an inequality.
Geometry and Measurement					
Range	7.GM.AV.1	Substitute correct values into the area and circumference formula.	Applies given formulas to calculate the area and circumference of a circle.	Describes the relationship between the circumference and diameter of a circle (pi) and applies given formulas to calculate the area and circumference of a circle, including authentic problems.	Describes the relationship between the circumference and diameter of a circle (pi) and calculates the missing dimension of a circle when given the area or circumference.
Range	7.GM.AV.2	Calculates areas of triangles and quadrilaterals by composing and/or decomposing them into rectangles and triangles and solves problems involving the surface area of cubes, right rectangular prisms and right triangular prisms using given nets-	Calculates areas of regular polygons by composing and/or decomposing them into rectangles and triangles, and solves problems involving the surface area of cubes and right prisms using given nets.	Calculates areas of polygons by composing and/or decomposing them into rectangles and triangles, including authentic problems and solves problems involving the surface area of prisms and right pyramids using nets, including authentic problems.	Calculates missing dimensions of polygons when given total area, including authentic problems and calculates missing dimensions of prisms and right pyramids when given the surface area, including authentic problems.
Range	7.GM.AV.3	Calculates the area of the base of cubes and right rectangular prisms.	Solves volume problems for right prisms.	Solve problems involving the volume of prisms and composite solids, including authentic problems.	Solves complex authentic problems involving the volume of prisms and composite solids and explains the relationships between a composite solid and the three-dimensional solids used to create it.
Range	7.GM.GF.1	Draws triangles from either three measures of angles or sides using appropriate tools.	Identifies whether a unique triangle, multiple triangles, or no triangle can be constructed when given three measures of angles or sides.	Draws triangles from given conditions using appropriate tools and defends whether a unique triangle, multiple triangles, or no triangle can be constructed when given three measures of angles or sides.	Draws triangles from given conditions using appropriate tools and explains how the triangle fits the given conditions and defends whether a unique triangle, multiple triangles, or no triangle can be constructed when given three measures of angles or sides, citing examples to support the defense.
Range	7.GM.GF.2	Identifies the following angle-pair relationships: supplementary angles, complementary angles, vertical angles, and adjacent angles.	Describes the following angle-pair relationships: supplementary angles, complementary angles, vertical angles, and adjacent angles, and solves for an unknown angle in a figure when angle relationships are stated.	Describes the following angle-pair relationships: supplementary angles, complementary angles, vertical angles, and adjacent angles, and solves for an unknown angle in a figure by applying facts about these angles.	Solves-for multiple unknown angles in a figure by applying facts about these angles and models the following angle-pair relationships: supplementary angles, complementary angles, vertical angles, and adjacent
Data, Probability, and Statistics					
Range	7.DPS.D.1	Identifies a sample and a population from a given scenario.	Identifies the strengths and weaknesses of a population sample without providing an explanation.	Identifies the strengths and weaknesses of a population sample including bias in the process of the data collection.	Identifies the strengths and weaknesses of a population sample including bias in the process of the data collection and draws conclusions about the usefulness of the sample.
Range	7.DPS.D.2	Draws inferences about a population using single random samples by using given measures of center for the numerical data set.	Draws inferences about a population using single and multiple random samples by using measures of centers.	Analyzes and draws inferences about a population using single and multiple random samples by using given measures of center and variability for the numerical data set.	Compares two populations using single and multiple random samples by using given measures of center and variability for the numerical data set.
Range	7.DPS.P.1	Determines the theoretical probability of a simple event.	Develops a probability model to find probabilities of simple theoretical events.	Develops a probability model to find probabilities of theoretical events and contrasts probabilities from an experimental model.	Compares and justifies the results from an experimental probability model and a theoretical probability model.

MATHEMATICS GRADE 7					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Range	7.DPS.P.2	Determines the sample space for compound events.	Determines the theoretical probabilities of independent compound events.	Develops a probability model to find theoretical probabilities of independent compound events.	Compares probability models to determine which best predicts the theoretical probabilities of independent compound events.



MATHEMATICS GRADE 8					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Policy		The Level 1 student has not yet reached the approaching proficiency level in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student consistently performs below the standard for the grade level/course, is inconsistently able to access grade-level content, and only engages with higher-order thinking skills with extensive support.	The Level 2 student is approaching proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student generally performs slightly below the standard for the grade level/course, is sometimes able to access grade-level content, and engages in higher-order thinking skills with some independence and support.	The Level 3 student is proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student regularly performs at the standard for the grade level/course, is able to access grade-level content, and engages in higher-order thinking skills with independence and minimal support.	The Level 4 student is highly proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student regularly performs independently or significantly above the standard for the grade level/course, is able to access above grade-level content, and engages in higher order-thinking skills independently.
		The Level 1 Student	The Level 2 Student	The Level 3 Student	The Level 4 Student
Number and Operations					
Range	8.NO.NS.1	Identifies rational and common irrational numbers (ex. square root of 2, pi, etc.) within the real number system and recognizes that real numbers can only be classified as either rational or irrational.	Classifies rational and irrational numbers within the real number system.	Compares and classifies real numbers within the real number system.	Explains the classification of real numbers within the real number system and provides examples.
Range	8.NO.NS.2	Determines which two perfect squares a radicand falls between and use this approximation to locate irrational numbers on a number line diagram.	Approximates a rational value for an irrational number and compares the size of irrational numbers written as square roots.	Uses rational approximations of irrational numbers to compare the size of irrational numbers, locates them on a number line diagram, and estimates the value of irrational expressions involving one operation.	Explains how to improve an approximation, explains how to compare the size of irrational numbers, and compares the value of irrational expressions involving one operation.
Range	8.NO.NS.3	Recognizes scientific notation and orders numbers written in scientific notation.	Recognizes scientific notation generated by technology, uses scientific notation to estimate very large and very small quantities, compares and orders numbers written in scientific notation.	Interprets scientific notation that has been generated by technology, uses scientific notation to represent very large or very small quantities, and compares and orders numbers in both scientific and standard notation.	Interprets and explains scientific notation generated by technology, explains how to use scientific notation to represent very large or very small quantities, and explains how to compare and order numbers in both scientific and standard notation.
Range	8.NO.O.1	Evaluates square roots of perfect squares up to 100.	Evaluates square roots of perfect squares up to 144 and cube roots of perfect cubes up to 125.	Evaluates square roots of perfect squares up to 225 and cube roots of perfect cubes up to 1000.	Explains how square roots and cube roots relate to each other and to their radicands.
Range	8.NO.O.2	Adds, subtracts, multiplies, or divides positive rational numbers using strategies or procedures.	Adds, subtracts, multiplies, or divides rational numbers using strategies or procedures.	Adds, subtracts, multiplies, and divides rational numbers using strategies or procedures.	Justifies the steps taken to add, subtract, multiply, and divide rational numbers.
Algebraic Reasoning					
Range	8.AR.EE.1	Uses exponents to rewrite expressions involving repeated multiplication, and uses repeated multiplication to rewrite expressions involving natural number exponents.	Identifies the relationship between repeated multiplication and the properties of integer exponents, and rewrites $(a^b)(a^c)$ as $a^{(b+c)}$ and $(a^b)^c$ as $a^{bc}$ .	Explains the relationship between repeated multiplication and the properties of integer exponents and applies a single exponent property to generate equivalent numeric and algebraic expressions that include numerical coefficients.	Applies two or more exponent properties to generate multiple equivalent numerical and algebraic expressions that include numerical coefficients and develops multiple justifications for properties of integer exponents.
Range	8.AR.EE.2	Identifies the index of square roots and cube roots.	Recognizes the number of real solutions to $x^2=p$ and $x^3=p$ , where $p$ is a non-negative integer.	Uses square root and cube root symbols to represent solutions to equations of the form $x^2=p$ and $x^3=p$ , where $p$ is a non-negative rational number.	Explains how square roots and cube roots relate to each other and to their radicands.
Range	8.AR.EE.3	Identifies the slope of a linear relationship in tables, graphs, equations, or descriptions.	Identifies the slope and y-intercept of a linear relationship in tables, graphs, equations, or descriptions.	Explains the characteristics of a linear relationship, including identifying the slope and y-intercept in tables, graphs, equations, and descriptions.	Compares and contrasts more than one linear relationship, including identifying the slopes and y-intercepts in tables, graphs, equations, and descriptions.
Range	8.AR.EE.4	Represents linear relationships using graphs or equations, when given a relationship in one of these forms.	Represents linear relationships using tables, graphs, and equations, when given a relationship in one of these forms.	Represents linear relationships using tables, graphs, equations, and descriptions when given a relationship in one of these forms.	Compare and contrast representations of different linear relationships, or different representations of the same linear relationship.

MATHEMATICS GRADE 8					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Range	8.AR.EE.5	Solves simple linear equations with rational coefficients.	Solves multi-step linear equations with rational coefficients and identifies equations that have one solution, infinitely many solutions, or no solutions.	Solves linear equations with rational number coefficients and variables on both sides, including equations that require using the distributive property and/or combining and collecting like terms, gives examples of linear equations in one variable that have one solution, infinitely many solutions, or no solutions, and interprets the number of solutions.	Justifies through multiple representations why an equation has one solution, infinitely many solutions, or no solutions including equations that require using the distributive property and/or combining and collecting like terms.
Range	8.AR.EE.6	Reads and evaluates numerical expressions including expressions involving absolute value.  Solves equations of the form $ x  = r$ where $r$ is a whole number.	Reads and evaluates numerical and algebraic expressions including expressions involving absolute value.  Solves and graphs equations of the form $ x  = r$ where $r$ is a whole number.	Reads, writes, and evaluates numerical and algebraic expressions including expressions involving absolute value.  Solves and graphs equations of the form $ x  = r$ where $r$ is a nonnegative rational number.	Justifies the steps taken to evaluate numerical and algebraic expressions including expressions involving absolute value.  Writes an equation from a graph of the form $ x  = r$ where $r$ is a nonnegative rational number.
Range	8.AR.EE.7	Solves and graphs inequalities in one variable with integer coefficients and variables on both sides.	Solves and graphs inequalities in one variable with integer coefficients and variables on both sides, including inequalities that require using the distributive property and/or combining like terms.	Solves and graphs inequalities in one variable with rational number coefficients and variables on both sides, including inequalities that require using the distributive property and/or combining like terms.	Justifies the steps to solve and graph inequalities in one variable with rational number coefficients and variables on both sides, including inequalities that require using the distributive property and/or combining like terms.
Range	8.AR.EE.8	Graphs linear inequalities in two variables on a coordinate plane.	Graphs linear inequalities in two variables on a coordinate plane and identifies the possible solutions.	Graphs linear inequalities in two variables on a coordinate plane and interprets the possible solutions in the context of authentic problems.	Writes linear inequalities in two variables from a graph on a coordinate plane and explains the possible solutions in the context of authentic problems.
Range	8.AR.F.1	Identifies whether a relation is a function from a graph or a mapping diagram.	Identifies whether a relation is a function from various representations using appropriate function language.	Defends whether a relation is a function from various representations using appropriate function language.	Creates any representation of a function and proves why it is a function using appropriate function language.
Range	8.AR.F.2	Given a representation of a linear function, creates another representation of that function.	Compares and contrasts properties of two linear functions each represented in the same way.	Compares and contrasts properties of two linear functions each represented in a different way (algebraically, graphically, numerically in tables, or verbal descriptions).	Justifies whether two linear functions represented in different ways are equivalent or not by comparing their properties.
Range	8.AR.F.3	Identifies whether a function is linear or non-linear.	Compares and contrasts linear and non-linear functions represented in the same way.	Compares and contrasts linear and non-linear functions represented in different ways (algebraically, graphically, numerically in tables, and/or by descriptions).	Defends which representation is preferred for comparing two given linear/nonlinear relationships and explains why the function is linear or nonlinear.
Range	8.AR.F.4	Determines the rate of change of the function from a graphical description of the linear function.	Determines the rate of change and initial value of a linear function from two $(x, y)$ values and creates a graph of a linear function.	Models a linear function between two quantities by creating a table, graph, and equation and interprets the rate of change and initial value of a linear function in terms of the situation it models.	Explains the relationship between two quantities in a linear function by analyzing a table, graph, and equation and interprets the rate of change and initial value of a linear function in terms of the situation it models.
Range	8.AR.F.5	Identifies when a graph of a functional relationship is constant, increasing, or decreasing; linear or nonlinear.	Describes qualitatively the functional relationship between two quantities by analyzing a graph including where the function is constant, increasing, or decreasing; linear or nonlinear and identifies a graph that exhibits the qualitative features of a function described.	Describe qualitatively the functional relationship between two quantities by analyzing a graph including where the function is constant, increasing, or decreasing; linear or nonlinear; and discrete or continuous and creates a graph that exhibits the qualitative features of a function described.	Compares two or more functional relationships between two quantities by analyzing graphs including where the functions are constant, increasing, or decreasing; linear or nonlinear; and discrete or continuous and creates two or more graphs that exhibit the qualitative features of two or more functions described.
Geometry and Measurement					
Range	8.GM.AV.1	Applies given formulas to solve problems involving the volume of cones, cylinders or spheres.	Applies given formulas to solve problems involving the volume of cones, cylinders, and spheres.	Applies given formulas to solve problems involving the volume of cones, cylinders, and spheres, including authentic problems.	Calculates a missing dimension when given the volume of cones, cylinders, and spheres, including in context of authentic problems.

MATHEMATICS GRADE 8					
Level	Standard	Novice	Approaching Proficient	Proficient	Advanced
Range	8.GM.GF.1	Identifies the corresponding lines, line segments, or angles in a single rigid transformation.	Performs single transformations to a figure on the coordinate plane.	Performs single transformations to a figure on the coordinate plane and determines whether the figures are congruent or similar.	Recognizes and explains how single transformations to a figure on the coordinate plane determine whether the figures are congruent or similar.
Range	8.GM.GF.2	Identifies a visual representation of a dilation, translation, rotation, or reflection on the coordinate plane using transformation language.	Describes the effect of reflections and translations on the coordinate plane using transformation language.	Describes the characteristics of transformations on the coordinate plane using transformation language.	Describes the characteristics of multiple transformations on the coordinate plane using transformation language.
Range	8.GM.GF.3	Recognizes when a single transformation is performed to map a pre-image to its image.	Recognizes when multiple transformations are performed to map a pre-image to its image.	Names the type of transformation(s) needed to map a pre-image to its image.	Explains the type of transformation(s) needed to map a pre-image to its image.
Range	8.GM.GF.4	Identifies angle pairs when parallel lines are cut by a transversal and finds angle measurements when given information about known angles.	Finds unknown angle measures in a triangle, and unknown angle measures for angle pairs when parallel lines are cut by a transversal.	Describes the following angle-pair relationships: interior and exterior angles of triangles and angles formed when a transversal cuts parallel lines or intersecting lines and solves for an unknown angle in a figure by applying facts about these angles.	Explains the following angle-pair relationships: interior and exterior angles of triangles and angles formed when a transversal cuts parallel lines or intersecting lines and solves for multiple unknown angles in a figure by applying facts about these angles.
Range	8.GM.GF.5	Identifies the leg lengths and the hypotenuse length of a right triangle.	Determines whether a triangle is a right triangle when provided the leg lengths and the hypotenuse length.	Describes the relationship between the leg lengths and the hypotenuse length of a right triangle and determines whether a triangle is a right triangle using this relationship.	Models the relationship between the leg lengths and the hypotenuse length of a right triangle using multiple representations and reasons why a triangle is or is not a right triangle using the relationship between leg lengths and the hypotenuse length.
Range	8.G.6	Calculates unknown hypotenuse side length given the Pythagorean Theorem.	Calculates unknown side lengths using the Pythagorean Theorem given two different side lengths of a right triangle in two dimensions on and off a coordinate plane.	Applies the Pythagorean Theorem to determine unknown side lengths in right triangles in two and three dimensions on and off a coordinate plane, including authentic problems.	Applies the Pythagorean Theorem in multi- step problems to determine unknown side lengths in right triangles in two and three dimensions on and off a coordinate plane, including authentic problems and finds the coordinates of a point which is a given distance (non-vertical and non-horizontal) from another point.
Data, Probability, and Statistics					
Range	8.DPS.D.1	Recognizes that scatter plots represent bivariate data.	Interprets scatter plots for bivariate measurement data and describes the pattern as positive, negative, or no relationship.	Interprets scatter plots for bivariate measurements data to investigate patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	Constructs scatter plots for bivariate measurement data to draw conclusions about clustering, outliers, positive or negative association, linear association, and nonlinear association.
Range	8.DPS.D.2	Recognizes a line can be used to describe a linear association on a scatter plot.	Draws an informal trend line on a scatter plot with a linear association.	Draws an informal trend line on a given scatter plot with a linear association and justify its fit by describing the closeness of the data points to the line.	Compares more than one trend line on a given scatter plot with a linear association and justifies the best one by describing the closeness of the data points to the line.
Range	8.DPS.D.3	Identifies the slope and y-intercept of a linear model for bivariate measurement data.	Identifies the slope and intercept(s) of a linear model for bivariate measurement data and makes predictions using the model.	Solves authentic problems in the context of bivariate measurement data by interpreting the slope and intercept(s) of a linear model and making predictions using the model.	Creates and uses a linear model based on a set of bivariate data by interpreting the slope and intercept(s) and making predictions using a linear model.
Range	8.DPS.D.4	Completes a partially filled-in two-way table summarizing bivariate categorical data collected from the same subjects.	Constructs a two-way table summarizing bivariate categorical data collected from the same subjects.	Constructs and Interprets a two-way table summarizing bivariate categorical data collected from the same subjects.	Constructs, interprets and compares relative frequencies to identify patterns of association.

MATHEMATICS GRADE 9-10				
Standard	Novice	Approaching Proficient	Proficient	Advanced
	The Level 1 student has not yet reached the approaching proficiency level in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student consistently performs below the standard for the grade level/course, is inconsistently able to access grade-level content, and only engages with higher-order thinking skills with extensive support.	The Level 2 student is approaching proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student generally performs slightly below the standard for the grade level/course, is sometimes able to access grade-level content, and engages in higher-order thinking skills with some independence and support.	The Level 3 student is proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student regularly performs at the standard for the grade level/course, is able to access grade-level content, and engages in higher-order thinking skills with independence and minimal support.	The Level 4 student is highly proficient in applying mathematics knowledge/skills as specified in the North Dakota Mathematics Content Standards. The student regularly performs independently or significantly above the standard for the grade level/course, is able to access above grade-level content, and engages in higher order-thinking skills independently.
	The Level 1 Student	The Level 2 Student	The Level 3 Student	The Level 4 Student
Number and Operations				
9-10.NO.1	Rewrites rational exponents using one property of exponents.	Rewrites simple expressions involving radicals or rewrites rational exponents using two properties of exponents.	Explains how the definition of rational exponents follows from extending the properties of integer exponents; rewrites simple expressions involving radicals and rational exponents using the properties of exponents.	Explains the method used in rewriting rational exponents and recognizes and identifies an error and shows the correct answer when rewriting rational exponents.
9-10.NO.2	Simplifies square roots of non-perfect squares and cube roots of non-perfect cubes.	Performs basic operations on radicals with like radicands and simplifies the radicals.	Performs basic operations on simple radical expressions to write a simplified equivalent expression.	Performs basic operations on radicals that include variables in the radicand, and simplifies radical expressions to write equivalent expressions.
9-10.NO.3	Identifies the scale or the units when given graphs and data displays.	Interprets the scale or the units when given graphs and data displays.	Chooses and interprets the scale and the units in graphs and data displays.	Explains why the chosen scale or units are best for graphs and data displays, and explains the interpretation of the scale or units.
9-10.NO.4*	Defines appropriate quantities for the purpose of descriptive modeling.	Defines appropriate units when given the appropriate quantities for the purpose of descriptive modeling.	Defines appropriate quantities and units for the purpose of descriptive modeling.	Explains why the chosen quantities or units are most appropriate for a given authentic context.
9-10.NO.5	Uses an indicated level of accuracy or precision when reporting quantities.	Chooses a level of accuracy or precision appropriate to limitations on measurement when given a measurement tool.	Chooses a level of accuracy or precision appropriate to limitations on measurement when reporting quantities.	Explains why a chosen level of accuracy or precision is appropriate to limitations on measurement, and explains whether accuracy or precision is more important in a given authentic context.
Algebraic Reasoning				
9-10.AR.1	Rewrites an expression by combining like terms and using the distributive property.	Rewrites a quadratic expression by applying one factoring technique.	Uses the structure of an expression (i.e., quadratic and exponential) to identify ways to rewrite it.	Explains how the structure of an expression lends itself to a specific method of rewriting the expression.
9-10.AR.2	Rearranges a formula using one step.	Rearranges a formula using more than one step, with scaffolding.	Rearranges formulas to isolate a quantity or variable(s) of interest using the same reasoning as in solving equations.	Chooses, with justification, which variable in a formula to isolate, and rearranges the formula to isolate that variable.
9-10.AR.3*	Creates a linear equation or inequality in one variable that models a given situation.	Creates a quadratic equation or inequality or exponential equation in one variable that models a given situation.	Creates equations and inequalities in one variable and use them to solve problems, including equations arising from linear, quadratic, and exponential functions.	Explains how to create equations and inequalities in one variable, explains why equations and inequalities in one variable represent given problems, and explains solutions to equations and inequalities in one variable.
9-10.AR.4*	Identifies the slope and y-intercept of a linear function that represents a context.	Graphs an equation to represent a linear or exponential relationship, and interprets the slope and y-intercept of a linear model in terms of a context.	Creates linear and exponential equations in two or more variables to represent relationships between quantities, and graphs equations on coordinate axes with appropriate labels and scales.	Writes and graphs an exponential equation to model a context, and interprets the base value, initial value, and vertical shift/asymptote of the function; writes and graphs a linear equation to model a situation, and interprets the slope and intercepts of the function.
9-10.AR.5	Identifies an appropriate next step for solving a linear equation.	Solves a linear equation with multiple steps, without justifying the steps involved in solving.	Justifies each step in solving a linear equation that may or may not have a solution.	Explains and justifies the steps in solving a linear equation by applying and naming the properties of equality, inverses, and identities.
9-10.AR.6	Solves linear equations and inequalities in one variable, when the unknown is on one side.	Solves linear equations and inequalities in one variable, when the unknown is on both sides.	Solves linear equations and inequalities, including compound inequalities, in one variable.	Solves linear equations and inequalities in one variable, including equations with coefficients represented by letters within an authentic context.
9-10.AR.7*	Finds an approximate solution to a system of linear equations when given a graph of the system.	Tests a potential solution to a system of linear equations by substituting values into each equation or by using graphs of the equations.	Solves a system of linear equations graphically and algebraically; creates and solves a system of linear equations in context.	Creates and analyzes a system of linear equations and solves exactly and approximately given a context or authentic situation, and manipulates one of the equations to provide additional information about the situation.
9-10.AR.8	Identifies a solution region when the graph of the boundary lines in a system of linear inequalities is given.	Graphs the solution set to a two-variable system of linear inequalities as the intersection of two half-planes.	Graphs the solution set to a two-variable system of linear inequalities; creates and graphs the solution set to a two-variable system of linear inequalities in context.	Writes or creates a two-variable system of linear inequalities given graph, and identifies the solution set as a region of the coordinate plane that satisfies all inequalities.



MATHEMATICS GRADE 9-10				
Standard	Novice	Approaching Proficient	Proficient	Advanced
9-10.AR.9	Solves absolute value equations in one variable.	Solves absolute value inequalities in one variable.	Solves absolute value equations and inequalities in one or two variables.	Explains and justifies the steps in solving absolute value equations or inequalities in one or two variables.
9-10.AR.10	Solves quadratic equations by simple inspection.	Solves quadratic equations that have rational number solutions.	Solves quadratic equations in one variable by inspection (e.g., for $x^2 = 49$ ) taking square roots, the quadratic formula, and factoring, as appropriate to the initial form of the equation.	Determines the most efficient method for solving a quadratic equation and justifies the choice selected.
9-10.AR.11	Performs one of the three operations on polynomials: adds, subtracts, or multiplies.	Performs two of the three operations on polynomials: adds, subtracts, or multiplies.	Adds, subtracts, and multiplies polynomials.	Adds, subtracts, and multiplies polynomial expressions in an authentic context.
9-10.AR.F.1	Identifies functions and non-functions based on equations, tables, graphs, or descriptions.	Determines whether a relationship is a function given a table, graph, or words, and determines the domain and range of a function.	Determines whether a relationship is a function given a table, graph, or words, identifying $x$ as an element of the domain and $f(x)$ as an element in the range, and determines the domain and range of a function in context.	Uses functions, function notation, graphs, and domain and range to represent and describe authentic contexts.
9-10.AR.F.2*	Evaluates simple functions at inputs in their domain, and uses $f(x)$ in an equation to represent the output of a function.	Evaluates functions for inputs in their domain and writes functions using function notation (without context).	Uses function notation, evaluates functions for inputs in their domains, and interprets statements that use function notation in context.	Creates context from a given domain and range and uses function notation to write an equation to model the context.
9-10.AR.F.3*	Identifies the key features of linear and exponential graphs, identifies the graph of linear and quadratic functions given their equations, and identifies the graph of an exponential function given its equation.	Interprets the key features of linear and exponential graphs, constructs the graphs of linear and quadratic functions given their equations, and constructs the graph of an exponential function given its equation.	Sketches the key features (to include intercepts, maximums, minimums, and lines of symmetry, where applicable) of linear, exponential, and quadratic functions modeling the relationship between two quantities using tables, graphs, written descriptions, and equations.	Creates graphs to model a situation, graphs and compares linear and quadratic functions expressed in various forms, and graphs exponential equations generated from real-life contexts.
9-10.AR.F.4*	Identifies the domain of a function when given a graph.	Relates the domain of a function to its graph, and graphs a function with a given restricted domain.	Relates the domain of a linear, quadratic, or exponential function to its graph and, where applicable, to the quantitative relationship it describes.	Creates a function for a given context where the domain meets given parameters.
9-10.AR.F.5*	Determines the average rate of change of a linear, quadratic, or exponential function (presented in a table) over a specified interval.	Determines the average rate of change of a linear, quadratic, or exponential function (presented algebraically) over a specified interval.	Calculates and interprets the average rate of change of linear, quadratic, and exponential functions (presented algebraically or as a table) over specified intervals, and estimates the rate of change from a graph.	Compares and interprets the rates of change of different types of functions.
9-10.AR.F.6*	Identifies properties of a linear, quadratic, or exponential function that are apparent in their given form (e.g., identifies the zeros of a quadratic function given in factored form).	Rewrites linear, quadratic, and exponential functions in different but equivalent forms.	Writes a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function. a. Uses appropriate forms of linear, quadratic, and exponential functions to show zeros, extreme values, and symmetry (where applicable) and interpret them in context. b. Uses the properties of an exponential function to classify it as growth or decay	Justifies which form of an expression is most valuable for revealing pertinent features in an authentic context, then rewrites and interprets the expression in context.
9-10.AR.F.7*	Compares key features of two functions of the same representation (e.g., a table to a table, or an equation to an equation) and compares the values of functions at specific points.	Compares key features of two functions of the same type with different representations (such as a linear to linear, but using a table and equation) and compares the values of functions over various intervals.	Compares key features of two linear, exponential, or quadratic functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	Constructs a linear or exponential function that has a characteristic (i.e., slope, $x$ - or $y$ -intercept, maximum) that is greater than or less than a given function and observes, explores, predicts, models, and evaluates different situations that compare linear, quadratic, and exponential functions.
9-10.AR.F.8*	Identifies situations in which one quantity changes at a constant rate per unit interval relative to another.	Uses tables or graphs to recognize relationships that can be modeled with linear functions and with exponential functions.	Identifies situations that can be modeled with linear, quadratic, and exponential functions, and justifies the most appropriate model for a situation based on the rate of change over equal intervals, including situations in which a quantity grows or decays.	Describes the rate of change per unit as constant or the growth factor as a constant percentage and proves that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals.
9-10.AR.F.9*	Identifies the effects of vertical translations on linear, absolute value, and quadratic graphs; and describes what will happen to a function when $f(x)$ is replaced by $f(x) + k$ .	Identifies the effects of horizontal and vertical translations on linear, absolute value, and quadratic graphs; describes what will happen to a function when $f(x)$ is replaced by $f(x) + k$ or $f(x - h)$ ; and finds the value of $h$ and $k$ given the graph of an absolute value or quadratic function.	Identifies the effect of transformations on the graph of a linear, absolute value, or quadratic function by replacing $f(x)$ with $af(x)$ , $f(x - h)$ , and $f(x) + k$ , for specific values of $a$ , $h$ , and $k$ (both positive and negative), and finds the value of $a$ , $h$ , and $k$ given the graph of the function.	Recognizes which transformations take away the even nature of a quadratic or absolute value function.

MATHEMATICS GRADE 9-10				
Standard	Novice	Approaching Proficient	Proficient	Advanced
9-10.AR.F.10	Finds the inverse of a linear function.	Describes the relationship between the domain and range of the function and its inverse without a context.	Finds the inverse of a linear function and describes the relationship between the domain, range, and graph of the function and its inverse in context.	Verifies experimentally that two functions are inverses by comparing input and output values.
9-10.AR-F.11*	Identifies the values of m and b for a linear function in slope-intercept form; identifies the values of a and c for a quadratic function in standard form; identifies the values of a and b for an exponential function in the form $f(x) = a(b)^x$ .	Describes the mathematical meaning of each parameter of linear (m and b), quadratic (a and c only), and exponential (a and b) functions.	Interprets the parameters in linear, quadratic, and exponential functions in context.	Interprets all parameters of a quadratic function written in standard or vertex form, and interprets all parameters of an exponential function written in the form $g(x) = a(b)^x + k$ .
9-10.AR.F.12	Identifies, using a given graph, the solution(s) to linear and exponential functions $f(x) = g(x)$ .	Identifies, by completing a given table, the solution(s) to linear and exponential functions $f(x) = g(x)$ .	Identifies, using graphs or tables, the solution(s) to linear and exponential functions $f(x) = g(x)$ as x-value(s) that result in equivalent y-values.	Uses technology to describe solutions using the knowledge that continuous lines and curves contain an infinite number of solutions.
Geometry and Measurement				
9-10.GM.1	Identifies an angle, circle, perpendicular line, parallel line, and line segment using proper notation.	Informally defines an angle, circle, perpendicular line, parallel line, and line segment using examples and non-examples.	Knows precise definitions and notations of angle, circle, perpendicular line, parallel line, and line segment based on the undefined notions of point, line, and plane.	Identifies real-life examples of an angle, circle, perpendicular line, parallel line, and line segment using precise definitions.
9-10.GM.2	Knows which transformations are rigid transformations.	Represents transformations in the plane.	Represents transformations in the plane; describes transformations as functions taking points in the plane as inputs and giving other points as outputs; and compares transformations that preserve distance and angle to those that do not (i.e., rigid versus non-rigid motion).	Writes rules for reflections across horizontal and vertical lines other than the x- and y-axes, rotations about a point other than the origin, and dilations from a point other than the origin.
9-10.GM.3	Distinguishes between rotations and reflections of a triangle, rectangle, parallelogram, trapezoid, or regular polygon that map each figure onto itself or another figure.	Identifies lines and points of symmetry of a triangle, rectangle, parallelogram, trapezoid, or regular polygon that map each figure onto itself or another figure.	Describes the rotations and reflections of a triangle, rectangle, parallelogram, trapezoid, or regular polygon that map each figure onto itself or another figure.	Identifies a triangle, rectangle, parallelogram, trapezoid, or regular polygon that satisfies a description of rotational symmetry or lines of symmetry.
9-10.GM.4	Identifies the characteristics of translations in angles, circles, perpendicular lines, parallel lines, and line segments.	Identifies the characteristics of rotations, reflections, and translations in angles, circles, perpendicular lines, parallel lines, and line segments.	Develops or verifies the characteristics of rotations, reflections, and translations in angles, circles, perpendicular lines, parallel lines, and line segments.	Justifies the effects and characteristics of transformations in angles, circles, perpendicular lines, parallel lines, and line segments.
9-10.GM.5	Performs a series of two transformations [rotation(s), reflection(s), or translation(s)] on a given geometric figure.	Chooses (from a list) a series of transformations that will carry a given geometric figure onto another.	Draws the image of a figure that has undergone a series of transformations [rotation(s), reflection(s), or translation(s)] using a variety of methods (e.g., graph paper, tracing paper, or geometry software).	Describes a series of two or more transformations that carries a given figure onto another, and explains how the order of a series of transformations is performed may result in different outcomes.
9-10.GM.6	Distinguishes between congruent and non-congruent shapes.	Identifies transformations of a given figure based on descriptions of rigid motion.	Predicts the effect of a specified rigid motion on a given figure using geometric descriptions of rigid motions, and determines whether two figures are congruent using the definition of congruence in terms of rigid motions	Justifies the congruence of two complex figures using properties of rigid motion.
9-10.GM.7	Recognizes corresponding parts of congruent triangles (angles and sides)	Uses correct notation to clearly state the correspondence between the parts of the congruent triangles that are being compared.	Uses the definition of congruence, based on rigid motions, to show two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent.	Justifies that two triangles are congruent by showing that all pairs of corresponding parts are congruent.
9-10.GM.8	Identifies which congruence theorem can be used to prove that two given triangles are congruent.	Writes the congruency statement for two congruent triangles.	Proves two triangles are congruent using the congruence theorems.	Explains why certain combinations of angle and side congruence (e.g., SSA) do not provide enough evidence for triangle congruence.
9-10.GM.9	Describes examples of theorems about lines and angles.	Determines the validity of statements within a given proof of a theorem about lines and angles.	Proves and applies theorems about lines and angles.	Applies theorems about lines and angles to solve complex, authentic problems.
9-10.GM.10	Describes examples of theorems about triangles.	Determines the validity of statements within a given proof of a theorem about triangles.	Proves and applies theorems about triangles.	Proves geometric relationships using theorems about triangles and applies theorems about triangles to solve complex, authentic problems.
9-10.GM.11	Describes examples of theorems about parallelograms.	Determines the validity of statements within a given proof of a theorem about parallelograms.	Proves and applies theorems about parallelograms.	Proves geometric relationships using theorems about parallelograms and applies theorems about parallelograms to solve complex, authentic problems.
9-10.GM.12	Uses geometric constructions to copy a segment and completes one- or two-step geometric constructions.	Uses geometric constructions to copy an angle, and identifies a geometric construction given a set of steps or an image.	Makes basic geometric constructions (e.g., segment, angle, bisectors, parallel and perpendicular lines) with a variety of tools and methods.	Applies geometric constructions to solve authentic problems.
9-10.GM.14	Performs dilations in the coordinate plane centered at the origin.	Calculates specific measures resulting from a dilation given a center and a scale factor.	Verifies experimentally and justifies the properties of dilations given by a center and a scale factor.	Applies properties of dilations to solve complex geometric and authentic problems.

MATHEMATICS GRADE 9-10				
Standard	Novice	Approaching Proficient	Proficient	Advanced
9-10.GM.15	Identifies corresponding sides and angles given similar figures with different orientations.	Uses correct notation to clearly state the correspondence between the parts of the similar triangles that are being compared.	Uses transformations to decide if two given figures are similar, and applies the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	Explains why a given set of transformations does or does not result in similar figures.
9-10.GM.16	Identifies which similarity theorem can be used to prove that two given triangles are congruent.	Writes the similarity statement for two similar triangles.	Proves similarity theorems about triangles.	Explains why similarity theorems prove triangles are similar.
9-10.GM.17	Finds the measures of sides and angles of congruent and similar triangles.	Uses congruence and similarity criteria to solve problems involving triangles.	Applies knowledge of congruence and similarity criteria for triangles to solve problems and to prove relationships in various geometric figures.	Explains the geometric reasoning used when solving authentic problems involving similar and congruent figures.
9-10.GM.18	Identifies the hypotenuse and the legs opposite from / adjacent to an indicated acute angle when given a right triangle.	Identifies the sine, cosine, and tangent ratios of acute angles when given right triangles.	Recognizes how the properties of similar right triangles allow the trigonometric ratios to be defined and determine the sine, cosine, and tangent of an acute angle in a right triangle.	Justifies why the sine, cosine, and tangent ratios of an acute angle in similar right triangles are equivalent.
9-10.GM.20*	Chooses the trigonometric ratio (sine, cosine, or tangent) that can be used to find an unknown side length or angle measure in a right triangle.	Solves an equation that includes a trigonometric ratio to find an unknown side length or angle measure in a right triangle.	Solves applied problems involving right triangles using trigonometric ratios, the Pythagorean Theorem, and special right triangles (30°-60°-90° and 45°-45°-90°).	Uses right triangle relationships to solve multi-step problems in an authentic context.
9-10.GM.22	Identifies special segments and angles in a circle: inscribed angle, tangent segment, etc.	Applies theorems about line segments and angles in circles to find unknown lengths or angles without a context.	Applies theorems about relationships between line segments and circles or angles and circles formed by radii, diameter, secants, tangents, and chords to find unknown lengths or angles.	Proves theorems about relationships between angles, line segments, and arcs in circles.
9-10.GM.25	Identifies the quantities represented in the formulas for arc length and area of sectors of circles.	Uses the formulas for arc length and area in sectors of circles when given a circle.	Explains and uses the formulas for arc length and area of sectors of circles	Uses formulas for arc length and areas of sectors of circles to explain geometric relationships.
9-10.GM.26	Converts between radians and degrees.	Identifies the radian measure of an angle given the length of the arc length and the radius of a circle.	Recognizes that the radian measure of an angle is the ratio of the length of the arc to the length of the radius of a circle.	Uses the measure of the central angle, the length of the arc, or length of the radius of a circle to find a missing quantity in a given circle.
9-10.GM.27	Uses slope criteria to determine whether two lines are parallel, perpendicular, or neither, when given the slopes of the lines.	Uses slope criteria to determine whether two lines are parallel, perpendicular, or neither, when given graphs or equations of the lines.	Develops and verifies the slope criteria for parallel and perpendicular lines, and applies the slope criteria for parallel and perpendicular lines to solve problems.	Creates the equation of a line parallel or perpendicular to a given line that passes through a given point in an authentic context.
9-10.GM.28	Uses coordinates and simple geometric theorems to algebraically solve problems given a visual representation on a coordinate plane.	Verifies simple geometric theorems algebraically using coordinates when given a visual representation on the coordinate plane.	Verifies simple geometric theorems algebraically using coordinates, and verifies algebraically, using coordinates, that a given set of points produces a particular type of triangle or quadrilateral.	Proves statements about geometric figures using coordinates and simple geometric theorems and constructs visual representations on the coordinate plane that meet given conditions for coordinates.
9-10.GM.29	Determines whether a given point is the midpoint of a given segment.	Determines the midpoint of a line segment on a coordinate plane.	Determines the midpoint or endpoint of a line segment using coordinates.	Finds the point on a directed line segment between two given points that partitions the segment in a given ratio.
9-10.GM.30*	Computes the perimeter of rectangles and right triangles given on a coordinate plane.	Computes the area of rectangles and right triangles given on a coordinate plane.	Computes perimeters of polygons and areas of triangles, parallelograms, trapezoids, and kites using coordinates.	Computes perimeters of polygons and areas triangles, parallelograms, trapezoids, and kites using coordinates in a contextual problem.
9-10.GM.31	Identifies and labels parts of a diagram that can be used to explain the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and a cone.	Determines one piece of information necessary to complete a partially given explanation of the derivation of the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone.	Explains derivations of the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone.	Identifies and corrects errors in the explanations of the derivations of the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone.
9-10.GM.32	Calculates the surface area of rectangular and triangular prisms and pyramids when all relevant dimensions are labeled on a diagram.	Calculates the surface area for prisms, cylinders, pyramids, cones, and spheres when given a labeled diagram.	Calculates the surface area for prisms, cylinders, pyramids, cones, and spheres to solve problems.	Calculates the surface area for prisms, cylinders, pyramids, cones, spheres, and composite figures to model authentic situations and finds missing dimensions in those figures given the surface area.
9-10.GM.33	Calculates the volume of rectangular and triangular prisms and pyramids when all relevant dimensions are labeled on a diagram.	Calculates the volume for prisms, cylinders, pyramids, cones, and spheres when given a labeled diagram.	Knows and applies volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems, including those involving composite figures.	Finds the volume of prisms, cylinders, pyramids, cones, spheres, and composite figures in a real-life context.
9-10.GM.34	Identifies the shapes of two-dimensional cross-sections of three-dimensional objects when an image is provided and the cross-sections are parallel or perpendicular to the base of the objects.	Identifies the shapes of two-dimensional cross-sections of three-dimensional objects when an image is not provided and the cross-sections are parallel or perpendicular to the base of the objects.	Identifies the shapes of two-dimensional cross-sections of three-dimensional objects and identifies three-dimensional objects generated by rotations of two-dimensional objects.	Identifies multiple shapes made from two-dimensional cross-sections of three-dimensional objects and explains how three-dimensional objects can be generated by rotations of two-dimensional objects.

MATHEMATICS GRADE 9-10				
Standard	Novice	Approaching Proficient	Proficient	Advanced
9-10.GM.35*	Applies concepts of density based on area and volume when given the values required for determining the density (e.g., determine density in grams per cubic meter when given mass and volume).	Applies concepts of density based on area and volume when given one of the values required for determining the density (e.g., determine density in grams per cubic meter when given mass and enough information to find volume).	Applies concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot)	Explains or justifies concepts of density based on area and volume in modeling situations.
9-10.GM.36*	Identifies geometric methods to solve design problems.	Applies geometric methods to solve design problems when given the explicit measurements needed for the geometric figures.	Applies geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; scaling a model).	Explains or justifies geometric methods used to solve design problems.



MATHEMATICS GRADE 9-10				
Standard	Novice	Approaching Proficient	Proficient	Advanced
Data, Probability, and Statistics				
9-10.DPS.1*	Identifies dot plots, histograms, and box plots for a given set of data.	Graphs numerical data on a given number line using dot plots, histograms, and box plots.	Represents data with plots on the real number line (dot plots, histograms, and box plots).	Determines and justifies which type of data plot on a real number line would be most appropriate for a set of data and identifies advantages and disadvantages of different types of data plots.
9-10.DPS.2*	Describes informally the center and spread of a single set of data or graph and identifies shape, center, and spread of a data set.	Compares informally the similarities or differences in shape, center, or spread between two graphs and identifies and states the effects of existing outliers.	Compares the center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets using statistics appropriate to the shape of the data distribution.	Plots data based on situations with multiple data sets, and then compares and discusses using measures of center and spread, justifies which measure(s) are most appropriate for comparison, identifies advantages and disadvantages of using each measure of center and spread, plots and interprets data based on contextual situations involving outliers, and then compares and discusses center and spread and explores the manipulation of additional data points.
9-10.DPS.3*	Determines whether data on a given scatter plot follow a linear trend.		Represents data on two quantitative variables on a scatter plot and describes how the variables are related. a. Fits a linear function to the data (with or without technology) if appropriate. b. Computes (using technology) and interprets the correlation coefficient of a linear fit. c. Interprets the meaning of the slope and y-intercept of the linear model in context. d. Interpolates and extrapolates the linear model to predict values.	Compares the fit of different functions to data and explains how to determine which function has the best fit.
9-10.DPS.4*	Identifies causation or correlation.	Defines causation and correlation.	Distinguishes between correlation and causation.	Justifies a claim of causation in an authentic example and demonstrates an understanding that a strong correlation does not imply causation.
9-10.DPS.5*	Identifies an event as a subset of a sample space.	Identifies the relationships between sets of events.	Describes events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes or as unions, intersections, or complements of other events (“or,” “and,” “not”).	Explains a set of outcomes in context using multiple representations.
9-10.DPS.6*	Determines whether events are dependent or independent, when no calculations are needed.	Calculates probabilities for events (including joint probabilities).	Recognizes that event A is independent of event B if the probability of event A does not change in response to the occurrence of event B, and applies the formula $P(A \text{ and } B) = P(A) \cdot P(B)$ given that events A and B are independent.	Contrasts several events in a sample space and determines if they are independent by calculating the event probabilities.
9-10.DPS.7*	Determines whether a given situation/calculation is represented by conditional probability.	Uses a formula to calculate conditional probabilities.	Recognizes that the conditional probability of an event A given B is the probability that event A will occur given the knowledge that event B has already occurred, and calculates the conditional probability of A given B and interpret the answer in context.	Identifies and interprets independence of events in contextual problems, using conditional probabilities.
9-10.DPS.8*	Calculates the probability of individual events.	Calculates the probability of the intersection of events.	Applies the formula $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ and interprets the answer in context.	Justifies the formula $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ and the interpretation of an answer derived from the formula in terms of a context.
9-10.DPS.9*	Identifies the difference between permutations and combinations.	Calculates a given permutation or combination.	Determines the number of outcomes using permutations and combinations in context.	Justifies choosing either a permutation or combination to determine the number of outcomes based on a specific given context.
9-10.DPS.10*	Explains the meaning of the data in a two-way frequency table.	Creates a two-way frequency table showing the relationship between two categorical variables.	Constructs and interprets two-way frequency tables of data for two categorical variables, and uses the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.	Interprets, identifies, and describes associations and trends using a two-way frequency table in a given context.